

Year 11 Linear Algebra Notes

Equations

An equation is a formula where some values are equal to some other values. That allows someone to calculate any value when given the other values. The unknowns are often called **variables** (because they vary) and are represented by letters in the equation.

For example: $\text{Area} = \pi r^2$ is an equation that allows us to calculate the area of a circle if we are given the radius.

Without an equals sign you do not have an equation.

A **linear** equation is when there are only two variables and neither of them is raised to any power. The standard form is written:

$$\text{value} = \text{rate} \times \text{variable} + \text{constant}$$

But the order of the terms can be rearranged, so that we can say $\text{value} = \text{constant} + \text{rate} \times \text{variable}$ without any effect on the maths.

If $H = 5d + 7$, then we could also say $H = 7 + 5d$

We can also swap sides, so $P = \frac{1}{2}g + 9$ is the same as $\frac{1}{2}g + 9 = P$

While it does not matter which order an equation is written, it is always $\text{rate} \times \text{variable}$ and the constant is never multiplied.

The rate and constant can be any number, including fractions and decimals.

If the constant or rate are negative (including subtractions) then the negative stays with them when the equation is rearranged.

If $Y = 1.2f - 8$, then it is also true that $Y = -8 + 1.2f$

If $K = 1.5 - 3g$, then it is also true that $K = -3g + 1.5$

Writing equations

This is mostly just a matter of identifying the rate and the constant and stating what the letter variable represents and putting them into:

$$\text{value} = \text{rate} \times \text{variable} + \text{constant.}$$

- The constant will be the minimum amount or the starting amount (when the variable = 0).
- The rate will be the charge for each unit of the variable. Often give by “for each metre” or “per hour” or similar in the question.
- Any letter or symbol can be used for the variable, but it usually helps to make it the starting letter of the variable name or whatever unit it is measured in.

When writing equations it is important to make all the units the same. One has to be particularly careful with money:

If a photocopy card is \$2 and 10 cents a page, then the formula is $\text{Cost} = 0.10 \times \text{pages} + 2.00$

If the rate is zero, the equation is of the form $y = \text{number}$, perhaps with a condition such as “for x less than 20” (or mathematically, $x < 20$).

Using equations

The value which the equation is written to give can be obtained by putting in the numbers and calculating.

$$\text{If } K = 5x + 20 \text{ then when } x = 3 \text{ we get } K = 5 \times 3 + 20 = 35$$

$$\text{If } P = 12 + 1.2k \text{ then when } k = 20 \text{ we get } P = 12 + 1.2 \times 20 = 36$$

Solving

More complicated problems need to be answered by algebraic solving, especially if you want Merit, though sometimes guess and check will give the answer.

The basic rule of solving is that each step simplifies the equation by removing terms. To remove a term ***you need to do the opposite to both sides of the equation.***

The order that this is done is:

- Expand any brackets and group all terms.
- Cancel out the variables so that they remain only on one side
- Cancel out any number on the side with the variable
- Divide by the number in front of the variable.

The answer can be checked by seeing if it works, or by looking at the matching graph.

$$\text{If } K = 0.2p + 12 \text{ and we want to find the value of } p \text{ that gives } K = 40.$$

$$40 = 0.2p + 12$$

$$40 - 12 = 0.2p \text{ (+ } 12 - 12)$$

$$28 = 0.2p$$

$$28 \div 0.2 = (0.2 \div 0.2) p$$

$$p = 140$$

$$\text{If } C = 5p + 2 \text{ and also } C = 3p + 14 \text{ they are both true when } C = C.$$

$$5p + 2 = 3p + 14$$

$$5p - 3p + 2 = (3p - 3p) + 14$$

$$2p + 2 = 14$$

$$2p \text{ (+ } 2 - 2) = 14 - 2$$

$$2p = 12$$

$$(2 \div 2) p = 12 \div 2$$

$$p = 6$$

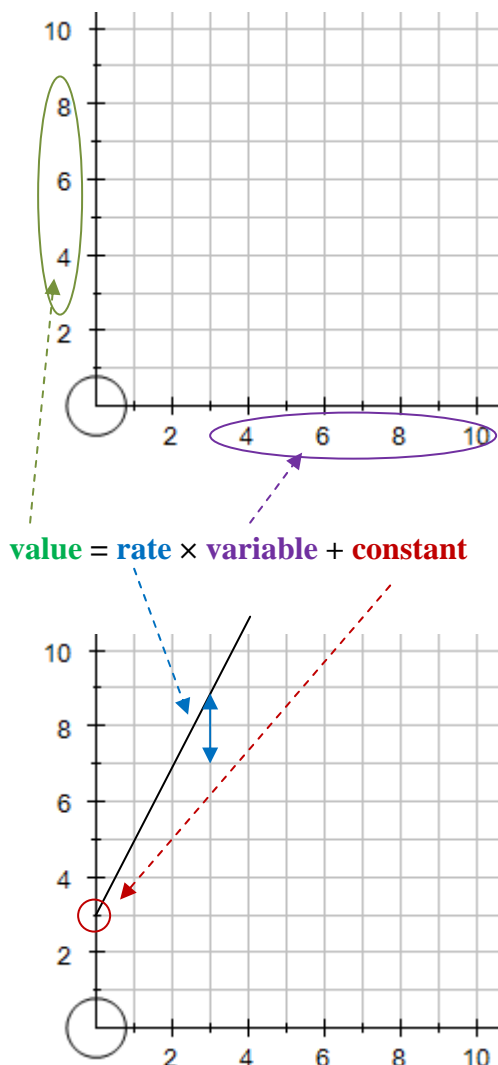
Care must be taken to ***answer the question in the context*** that it is asked.

- Sometimes decimal or fraction answers make no sense and must be rounded up or down.
- If the question is asked in terms of when is a value greater or less than something, then the answer must reflect that.

Graphs

Linear equations always give a line when drawn on a standard co-ordinate axis system. Normally both axes start at zero (even if this makes no sense in terms of the real life context) as this makes drawing the lines and checking them against the equations much easier.

The value that the equation is written to give goes on the y axis (vertical) whereas the variable in the equation goes along the x axis (horizontal).



When a line is drawn representing the equation, it starts at the value of the constant in the equation, and goes up (down if negative) by the amount of the rate for every one unit across on the x axis.

Drawing lines

To draw a line: plot a few known points from using your equation, taking care to get the right axes. If done correctly they should form a straight line, with the start point being the constant in the equation, and the rate in the equation being the slope of your graph.

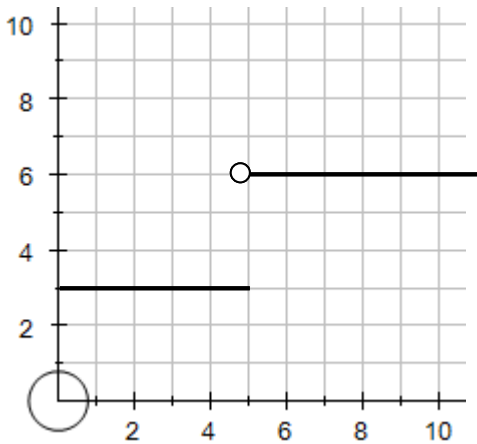
It does not matter which values are entered into the equations. Generally it is best to use round numbers to make plotting the points easier. If you already have some calculated values it makes sense to use those.

The more points you calculate the more likely you are to spot a mistake. The wider your points are apart, the more accurate your line will be.

If a value goes off your grid you cannot use it. Select another point.

If the equation is a constant with no rate component (e.g. $K = 30$) then the line is horizontal.

Occasionally a graph will be given in pieces, in which case each section is drawn separately. Sometimes those pieces may not even connect.



This is the graph of
 $y = 3$ for $x \leq 5$
and
 $y = 6$ for $5 < x$

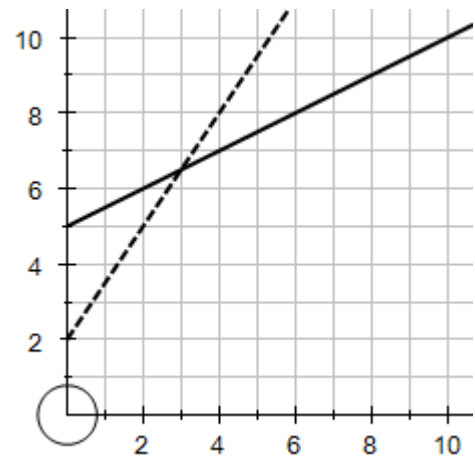
Using a graph

Graphs can be used to solve problems.

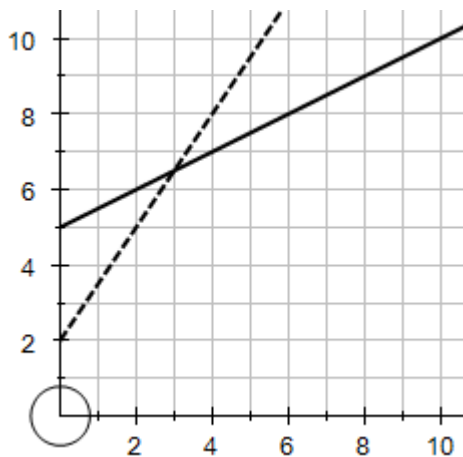
We can see that for $Y = 1.5x + 2$ (dotted line) that for Y to be 8, x must be 4.

We can also see that $Y = 1.5x + 2$ crosses $Y = 0.5x + 5$ (solid) when $x = 3$.

Lines cross when they are both true in the real world context, and gives the answer for the “simultaneous equation”.



Graphs can also be used to find ranges for which equations give the largest or smallest values for a particular value of the variable. Often that will have a useful meaning.



The dotted line of $y = 1.5x + 2$ is below the line of $y = 0.5x + 5$ for $x < 3$ (x is less than 3).

This means $1.5x + 2$ is less than $0.5x + 5$ for any value of x under 3.

Likewise, for values of x greater than 3, we can see that $1.5x + 2$ is greater than $0.5x + 5$.