Merit+ Circle Geometry Practice #1

1. Find \( \angle DBA \) (marked \( \theta \)).

2. Find \( \angle OZY \) (marked \( \theta \)).

3. Show that ST is parallel to QR.

4. JK is a tangent intersecting at L. If \( \angle LPN = 2 \times \angle MLK \) show that LM is the same length as MN.
**Answers: Merit+ Circle Geometry Practice #1**

1. **Find \( \angle DBA \)** (marked \( \theta \)).
   
   \[ \angle ADC = 90^\circ \] (subtended from ends of a diameter)
   \[ \angle DCA = 42^\circ \] (angles in triangle add to 180°)
   \[ \angle DBA = 42^\circ \] (angles subtended by the same arc are equal)

2. **Find \( \angle OZY \)** (marked \( \theta \)).
   
   \[ \angle YZX = 111^\circ \] (opposite sides of cyclic quadrilaterals add to 180°)
   \[ \angle OZX = \angle OXZ \] (base angles of an isosceles triangle are equal)
   \[ \angle OZX = 61^\circ \] (interior angles of a triangle add to 180°)
   \[ \angle OZY = 50^\circ \] (difference between \( \angle XZY \) and \( \angle OZX \))

3. **Show that ST is parallel to QR.**
   
   Let \( x = \angle QOR = \angle TOS \) (vertically opposite angles are equal)
   \[ \angle OST = \angle OTS \] (base angles of an isosceles triangle are equal)
   \[ \angle OST = 90^\circ - \frac{1}{2}x \] (interior angles of a triangle add to 180°)
   \[ \angle OQR = 90^\circ - \frac{1}{2}x \] (as for the triangle OTS above)
   \[ \angle OQR \text{ and } \angle OST \text{ are alternate and equal} \]
   
   ST must be parallel to QR as alternate angles on a transversal are equal.

4. **JK is a tangent intersecting at L.**
   
   If \( \angle LPN = 2 \times \angle MLK \) show that
   
   LM is the same length as MN
   
   Let \( 2x = \angle LPN \) so \( x = \angle MLK \)
   \[ \angle MLO = 90^\circ - x \] (Tangent is at 90° to a radius it touches)
   \[ \angle LMN = 180^\circ - 2x \] (opposite sides of cyclic quad add to 180°)
   \[ \angle LON = 4x \] (angle subtended to centre is twice the angle to the sides and \( \angle LPN = 2x \))
   \[ \angle ONM = 90^\circ - x \] (quadrilateral LMNO’s interior angles add to 360°)
   
   Since \( \angle MNO = \angle MLO \) and \( OL = ON \) (both radiuses) quadrilateral LMNO is symmetric
   
   LM and MN must be the same length if LMNO is symmetric.