Merit+ Circle Geometry Practice #3

1. Find \( \angle AOD \) (marked \( \theta \)).

2. Find \( \angle WZY \) (marked \( \theta \)).

3. Show that \( \angle QRT \) is half the size of \( \angle QOU \).

4. JM is a tangent intersecting at N. LO is 3 units long. KO is five units long. Find the length of JM.
Answers: Merit+ Circle Geometry Practice #3

1. Find $\angle AOD$ (marked $\theta$).

- $\angle ABO = 35^\circ$ (triangle formed by radii is isosceles)
- $\angle AOB = 110^\circ$ (interior angles of a triangle add to $180^\circ$)
- $\angle BOD$ reflex = $2 \times 130^\circ = 260^\circ$ (angle subtended to centre is $2 \times$ the angle to the sides)
- $\angle AOD = 150^\circ$ ($\angle BOD$ reflex $- \angle AOB$)

2. Find $\angle WZY$ (marked $\theta$).

- $\angle WYX = 21^\circ$ (base angles of an isosceles triangle are equal)
- $\angle WXY = 138^\circ$ (interior angles of a triangle add to $180^\circ$)
- $\angle WZY = 42^\circ$ (opposite angles of cyclic quad add to $180^\circ$)

3. Show that $\angle QRT$ is half the size of $\angle QOU$.

Let $\angle QRT = x$

- $\angle OQR = 90^\circ$ (tangent and radius from intersection are at $90^\circ$)
- $\angle QTR = 90^\circ - x$ (interior angles of a triangle add to $180^\circ$)
- $\angle QUS = 90^\circ - x$ (angles to edge subtended by same arc are equal)
- $\angle UQO = 90^\circ - x$ (base angles of isosceles triangle are equal)
- $\angle QOU = 2x$ (interior angles of a triangle add to $180^\circ$)
- $\angle QOU$ is $2 \times \angle QRT$

4. JM is a tangent intersecting at N.

LO is 3 units long. KO is five units long.

Find the length of JM.

- NO = 3 because it is the same size as LO (both radiuses)
- MO = 5 because it is the same size as KO
- $\angle MNO = 90^\circ$ (tangents are at $90^\circ$ to a radius at intersection)
- MNO is a right angle triangle, so we can use Pythagoras’ Theorem
  - $NM = \sqrt{5^2 - 3^2} = 4$
  - JM = $2 \times NM = 8$ units long