1. Find $\angle DEA$ (marked $\theta$).

2. Show that for every isosceles trapezium a circle can be drawn which goes through all four vertices.

   *(An isosceles trapezium is one where the non-parallel sides are of equal length.)*

3. A ball of radius 5 cm just fits inside a cone of 60° base as shown. What is the vertical height of the cone, $h$?

4. Two circles of different sizes intersect at $J$ and $K$. From $J$ two diameters are drawn, to $L$ and $M$. Show that $LKM$ is a straight line.
Answers: Merit+ Circle Geometry Practice #4

1. Find $\angle DEA$ (marked $\theta$).

$\angle CDB = 12^\circ$ (angles subtended by the same arc are equal)
$\angle CED = 121^\circ$ (angles in triangle add to $180^\circ$)
$\angle DEA = 59^\circ$ (angles on a line add up to $180^\circ$)

2. Show that for every isosceles trapezium a circle can be drawn which goes through all four vertices.

(An isosceles trapezium is one where the non-parallel sides are of equal length.)

Let $\angle XYZ = x$
$\angle YXW = 121^\circ - x$ (co-interior on $||$ add to $180^\circ$)
$\angle XWZ = 121^\circ - x$ (by symmetry, reflected in dotted mirror line)
$\angle XYZ + \angle XWZ = 180^\circ$ \Rightarrow the trapezium is a cyclic quadrilateral
A cyclic quadrilateral has all four vertices on a circle.

3. A ball of radius 5 cm just fits inside a cone of $60^\circ$ base as shown. What is the vertical height of the cone, $h$?

$h = 5 + x$ (radius of ball up + CR, distance down)
$\triangle CUR$ is right angle at $U$, as side $RT$ is tangent to ball
$x = 5 \div \sin(30^\circ)$ (trig, with $CU = 5$, as it is a radius)
$h = 5 + 10 \Rightarrow$ the height is 15 cm

4. Two circles of different sizes intersect at J and K.
From J two diameters are drawn, to L and M.
Show that LKM is a straight line.

Draw in line JK
$\angle JKL = 90^\circ$ (angle subtended by diameter is $90^\circ$)
$\angle JKM = 90^\circ$ (angle subtended by diameter is $90^\circ$)
$\angle JKL + \angle JKM = 180^\circ$
So LKM must be straight.