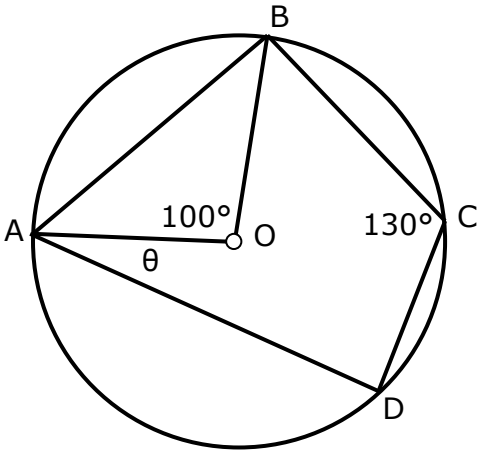
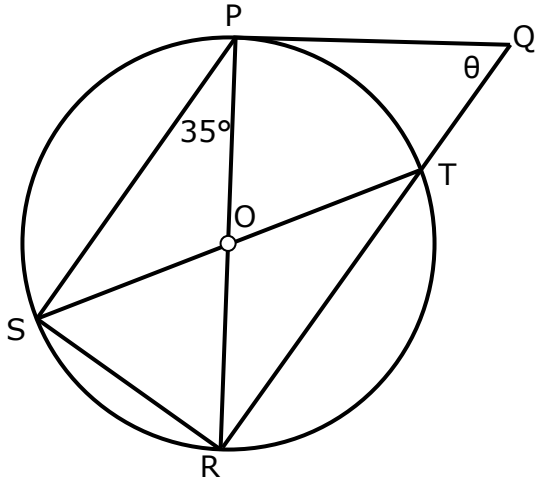
Merit+ Circle Geometry Practice #6

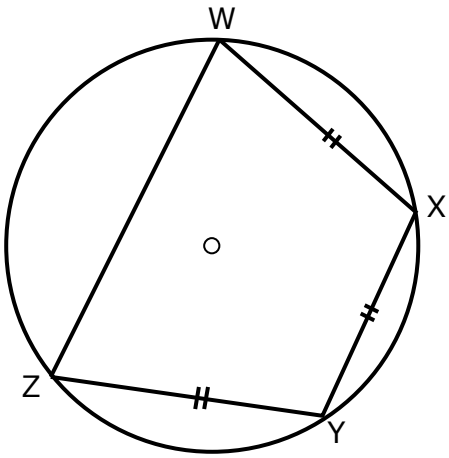
1. Find ∠OAD (marked θ).



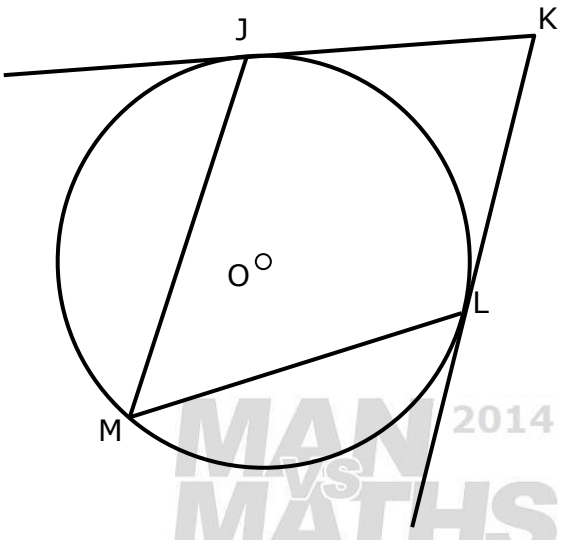
2. PQ is a tangent to the circle.
   Find ∠PQR (marked θ).



3. Lengths WX = WY = YZ.
   Show that WZ is parallel to XY

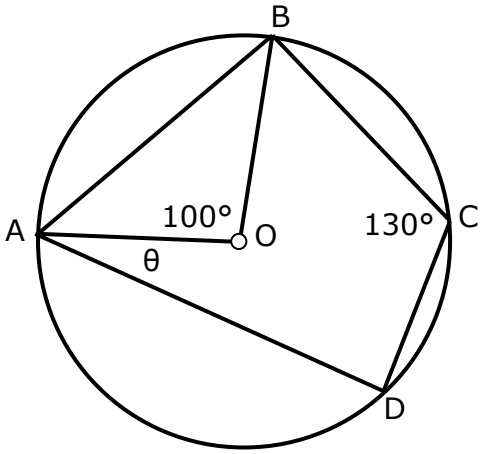


4. JK and KL are tangents intersecting at J and L.
   How large is ∠JKL, in terms of ∠JML.



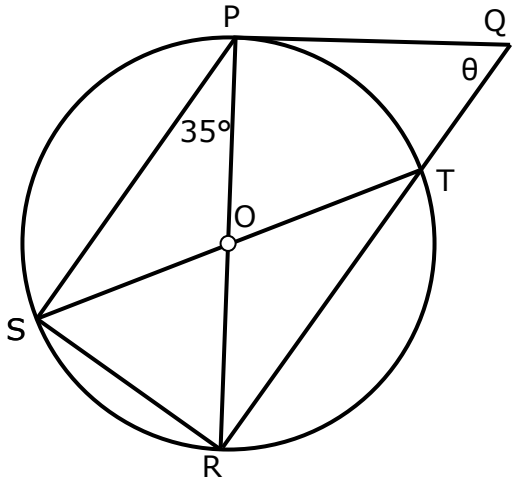
Answers: Merit+ Circle Geometry Practice #6

1. Find ∠OAD (marked θ).



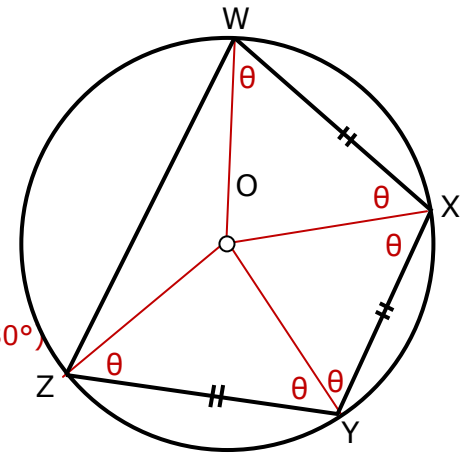
∠ABO = ∠ABO (triangle formed by radii is isosceles)
∠ABO = 40° (interior angles of a triangle add to 180°)
∠BAD = 50° (opposite angles cyclic quad add to 180°)
∠OAD = 10° (remainder when ∠BAO taken from ∠BAD)

2. PQ is a tangent to the circle.
Find ∠PQR (marked θ).



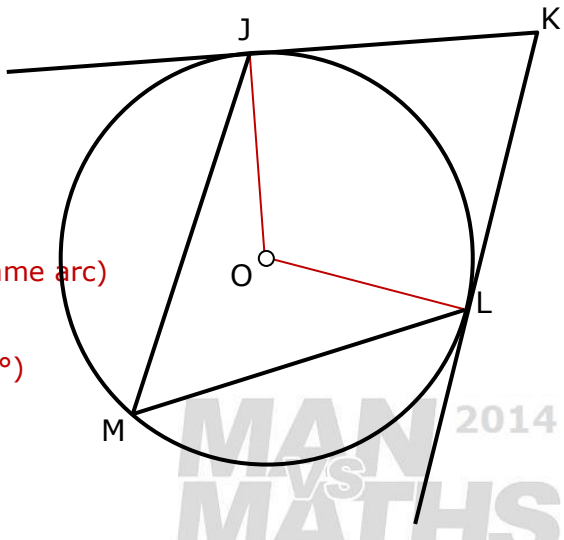
∠STR = 35° (angles from same arc to edge are equal)
∠PRT = 35° (interior angles of a triangle add to 180°)
∠RPQ = 90° (tangent and radius are at 90°)
∠PQR = 55° (interior angles of a triangle add to 180°)
or calculate ∠POS using isosceles Δ
then vertically opposite = ∠ROT
calculate ∠ORT using isosceles Δ etc

3. Lengths WX = WY = YZ.
Show that WZ is parallel to XY



Radii make three identical triangles (same base length and same sides = all identical angles too).
Let ∠WXO = θ, from which all same size are shown.
∠WZY = 180° − 2θ (opposite angles in cyclic quad add to 180°)
∠XYZ = 2θ so ∠XYZ + ∠WZY = 180°
Angles ∠XYZ and ∠WZY are co-interior and add to 180°
⇒ WZ and XY must be parallel.

4. JK and KL are tangents intersecting at J and L.
How large is∠JKL, in terms of ∠JML.



Let ∠JML = x
∠JOL = $2x$ (angle at centre is twice angle at edge from same arc)
∠KJO = ∠KLO = 90° (both tangents to radii)
∠JKL = 180 − $2x$ (interior angles quadrilateral add to 360°)
∠JKL = 180 − 2 × ∠JML