Trial Linear Algebra #2

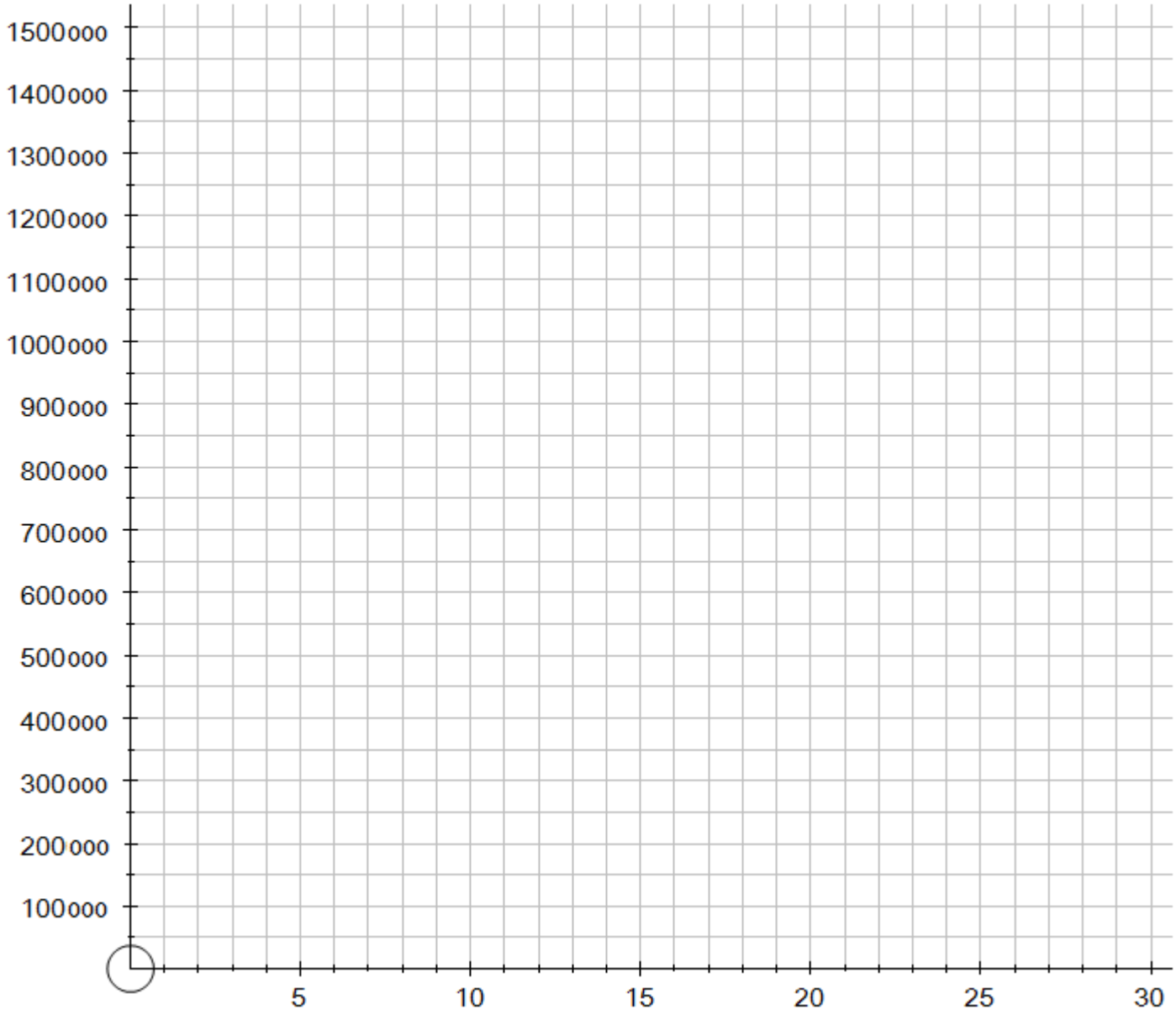
Three old fellers are thinking about retiring:

- Denis has $1,500,000 saved, and reckons he will need to spend about $50,000 a year.
- Eric has $500,000 saved and thinks he needs only $20,000 a year.
- Freddie has $700,000 saved, and estimates he needs to spend $40,000 a year.

a) Calculate how much each will have in twenty year's time if they all retired on what they have now.

b) Write equations for the different amounts on the basis of the number of years retired.

c) Graph the equations and use it to show when they will run out of money.



d) Advise Freddy how much he needs to spend each year if he is to afford 30 years of retirement. Show graphically how one calculates this.

e) Show graphically the result of Eric starting off with $100,000 more than he actually has at the start of his retirement.

f) When does Eric start having more money than Freddie?

Answers: Trial Linear Algebra #2

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a) Calculate how much each will have in twenty year's time if they all retired on what they have now.

Dennis = 1,500,000 – 20 × 50,000 = 500,000

Eric = 500,000 – 20 × 20,000 = 100,000

Freddie = 700,000 – 20 × 40,000 = –100,000 = no money left

b) Write equations for the different amounts on the basis of the number of years retired.

Dennis: Savings = 1500,000 – 50,000 × Years or $ = 1500,000 – 50,000 Y

Eric: Savings = 500,000 – 20,000 × Years or $ = 500,000 – 20,000 Y

Freddie: Savings = 700,000 – 40,000 × Years or $ = 700,000 – 40,000 Y

c) Graph the equations and use it to show when they will run out of money.

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d) Advise Freddy how much he needs to spend each year if he is to afford 30 years of retirement. Show graphically how one calculates this.

70,000 ÷ 30 = $23,333 per year

On the graph the green dotted line shows the new version. It starts out at the same point, but must end at the 30 year mark. For 10 years across it has dropped 233,333 on the graph, so it must be falling $23,333 each year.

e) Show graphically the result of Eric starting off with $100,000 more than he actually has at the start of his retirement.

The graph is shown dotted in purple, starting at $100,000 higher, and with exactly the same slope. That shows that he will now be able to last 30 years (five more).

f) When does Eric start having more money than Freddie?

On the graph we can see their lines cross at 10 years, and after that point Freddie's is lower than Eric's, meaning he has less money.

Solving it using Algebra:

500,000 – 20,000 Y = 700,000 – 40,000 Y

500~~,000~~ – 20,~~000~~ Y = 700,~~000~~ – 40,~~000~~ Y

500 – 20 Y + 40 Y = 700

20 Y = 700 – 500

20 Y = 200

Y = 200 ÷ 20 = 10

So they will have an equal amount after 10 years, and Freddie will have less after that.