Trial Measurement #2 – "Skate Park"

Useful formulas:

Area of a parallelogram = b h Area of a triangle = ½ b h

Area of a circle = πr^2 Circumference of a circle = π d

Volume of a prism, including cuboid or cylinder = base area × height

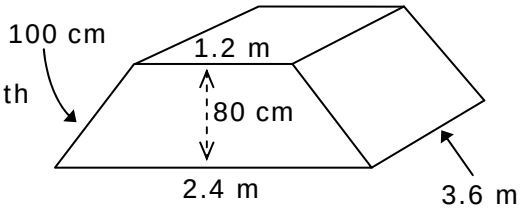
Volume of a pyramid or right cone = $\frac{1}{3}$ base area × height

Surface area of a pyramid = base area + ½ perimeter × slant height.

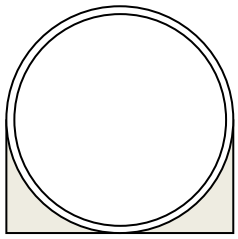
Surface area of a right cone = base area + ½ circumference × slant height

Your local town is building a new skate park. They plan to start with three new items.

The first is a symmetrical trapezoidal prism shaped with the dimensions shown opposite.



The next is to have a tube of concrete (already purchased) set in concrete along its entire length to hold it. A side on view is shown to the right.

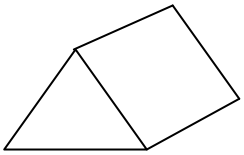


The tube has an outer diameter of 3.6 metres, an inside diameter of 3.4 metres and is 4.8 metres long.

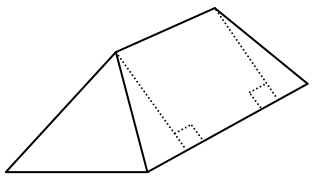
The concrete goes half way up the tube and is as wide as the outside of the tube (3.6 m).

They have asked you to design the third item. It must be:

- between 90 and 120 centimetres high vertically,
- at least as wide at the ground as it is high,
- at least 2 metres long,
- It should use about 1.5 m^3 of concrete.



The plan is to have it based on a triangular prism, preferably with sloping ends as well. The simple and the better options are shown on the right.



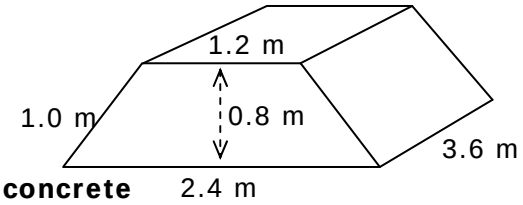
Each item in the park will be cast in concrete. The amount of concrete to order will be required in cubic metres. (Do not include the concrete tube as it is already bought).

Each item in the park will be coated on the visible surfaces with expensive, special anti-graffiti paint. Each visible surface will need painting – tops and sides but not the bases. The visible parts of the concrete tube will also need painting. A litre of paint will cover 5 m^2 of concrete.

Your task

Calculate the volume of concrete and the amount of paint you need to order. Do this separately for each item.

Answers: Trial Measurement #2 – "Skate Park"

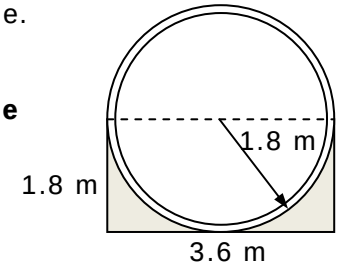


Convert all units to the same type, as shown.

Side trapezium =(2.4 + 1.2) ÷ 2 × 0.8 = 1.44 m^2
(or as rectangle + triangle = 1.2 × 0.8 + ½ × 1.2 × 0.8)

Volume = base area × depth = 1.44 × 3.6 = **5.184 m^3 of concrete**

Surface area = front + back + two sides + top (not base)
= 1.44 + 1.44 + 2 × (3.6 × 1.0) + 1.2 × 3.6 = 14.4 m^2
At 1 L for each 5 m^2, that requires 14.4 ÷ 5 = **2.9 Litres of paint**.

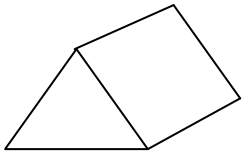


The area of the end concrete shown shaded = rectangle – half circle.
= 3.6 × 1.8 – ½ × π × 1.8^2 = 1.39 m^2

Volume = base area × depth = 1.39 × 4.8 = **6.672 m^3 of concrete**

Surface area = two concrete ends + two tube ends + two straight sides + curved outside top of tube + inside of tube.

= 2 × 1.39 + 2 × (π × 1.8^2 – π × 1.7^2) + 2 × 1.8 × 4.8
+ ½ × (π × 3.6 × 4.8) + π × 3.4 × 4.8
= 2.78 + 2.20 + 17.28 + 27.14 + 51.27 = 100.67 m^2
At 1 L for each 5 m2, that requires 100.67 ÷ 5 = **20.1 Litres of paint**.

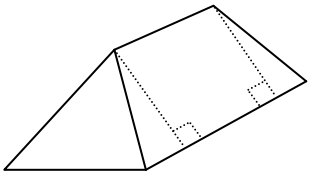


For a simple triangular prism, assume a height of 1 m and a width of 1.2.
This gives an end area = ½ × 1 × 1.2 = 0.6 m^2
so length needs to be about 1.5 ÷ 0.6 = 2.5 metres long.

A suitable size is 1.0 m high by 1.2 m wide by 2.5 m long. (Note this is only one possible solution, and other sizes give acceptable answers.)

The long edge needs Pythagoras = $\sqrt[2]{1.0^2 + 0.6^2}$ = 1.17 m

Surface area = front + back + 2 sides = 0.6 + 0.6 + 2 × 1.17 × 2.5 = 7.05 m^2
At 1 L for each 5 m2, that requires 7.05 ÷ 5 = **1.4 Litres of paint**.



When calculating a suitable sloping end version, the shape can be divided (where the dotted lines come down) into a triangular prism in the middle and two halves of a square pyramid at each end.

If I make the object 1 m high and 1.2 wide across the cross section base, the volume of the pyramid part is ⅓ of the matching cuboid = ⅓ × 1.2^2 × 1.0 = 0.48 m^3

That leaves 1.02 m^3 concrete for the rest of the prism. The cross section face is again 0.6 m^2, so the length needs to be = 1.02 ÷ 0.6 = 1.7 m long. (1.7 + 1.2 = 2.9 m overall)

I can make it **1 metre high, 1.2 across at the base, 2.9 m long and sloping in at the ends in at the same angle as the sides**. (Again other dimensions meet the criteria.)

Slant height needs Pythagoras = $\sqrt[2]{1.0^2 + 0.6^2}$ = 1.17 m

Surface area = pyramid sides + two sides (ends of triangular covered by pyramid part)
= 4 × ½ × 1.2 × 1.17 + 2 × 1.17 × 1.7 = 7.46 m^2 7.46 ÷ 5 = **1.5 Litres of paint**