L2 Calculus Practice #5

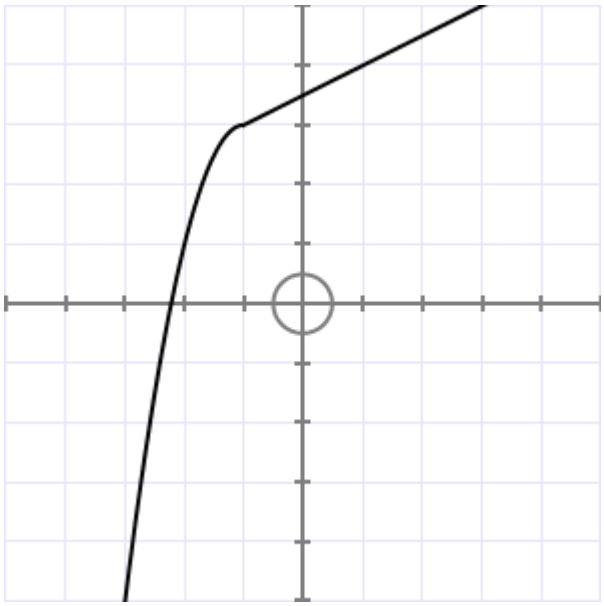
1. Find the gradient of the tangent to $y = 5 - 2x^2 + 4x$ at (2, 5)

2. A curve has $f(2) = 5$ and $f'(x) = 6x^2 - 3x$. Find the equation of the curve

3. Find the turning point(s) of the function given by $f(x) = 5x - 2x^2$

4. To the right is a **piecewise** function.

   Sketch the matching gradient function.



5. What is the maximum speed a car will go at if velocity (in km per hour) is given by the formula: $v = 10 + 8t - 0.2t^2$ (from $t = 0$ seconds, until the car stops)

6. Find the equation of the line with a gradient of 6 which is a tangent of $y = x^2 - 2x - 3$

Answers: L2 Calculus Practice #5

1. $y = 5 - 2x^2 + 4x$ so $\frac{dy}{dx} = -4x + 4$ at $x = 2$ $\frac{dy}{dx} = -4 \times 2 + 4 = -4$

   Gradient at (2, 5) = **–4**

2. $f'(x) = 6x^2 - 3x$ so $f(x) = 2x^3 - 1.5x^2 + C$

   told $f(2) = 5$ so $5 = 2 \times 2^3 - 1.5 \times 2^2 + C$ So $C = -5$

   Equation is $\boldsymbol{f(x) = 2x^3 - 1.5x^2 - 5}$

3 $f(x) = 5x - 2x^2$ so $f'(x) = 5 - 4x$ Turns where gradient = 0

   so $0 = 5 - 4x$ $x = 1.2$ Put this back into original equation

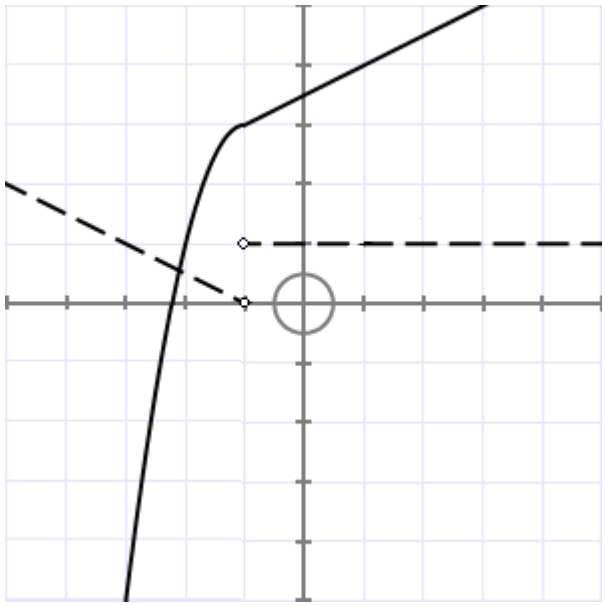
   $f(x) = 5 \times 1.2 - 2 \times 1.2^2$ **Turning point at (1.2, 3.12)**

4. **A line of negative slope** reaching zero (or close) at the point the pieces meet.

   A horizontal line of some positive value to the right of the join.

   The two gradient lines should **not** join.

   There is no gradient where the pieces of the original graph join. Showing this with empty circles at the ends of the lines is the correct method (but will not be insisted on).



5. $v = 10 + 8t - 0.2t^2$ so $\frac{dv}{dt} = 8 - 0.4t$

   max when $\frac{dv}{dt} = 0$ when $0 = 8 - 0.4t$ so at $t = 20$

   Asks for max speed, so find v at $t = 20$. $v = 10 + 8 \times 20 - 0.2 \times 20^2$ = **90 kph**

6 $y = x^2 - 2x - 3$ so $\frac{dy}{dx} = 2x - 2$ Need when gradient = 6

   so $\frac{dy}{dx} = 6 = 2x - 2$ so when $x = 4$ At point $(4, 4^2 - 2 \times 4 - 3)$

   using $y - y_1 = m(x - x_1)$ with gradient 6, through (4, 5) $y - 5 = 6(x - 4)$

   Equation of tangent is: $y = 6x - 19$

Questions 5 and 6 are Merit