Basic for Exponents

❶ Any variable can be written as itself to power 1. Any variable to power 0 is equal to 1.

$x = x^1$ $\qquad$ $x^0 = 1$

❷ Exponents can only be added if they are the same unknowns, raised to the same powers. The power does not change.

$4xy^2 + 3xy^2 = 7xy^2$ $\qquad$ but $y^2 + y^3$ cannot be simplified

❸ When exponents are multiplied, the result is the variable to the two exponents added.

$x^4 \times x^3 = x^{4+3} = x^7$

❹ When exponents are divided the result is the first exponent minus the second.

$x^5 \div x^2 = x^{5-2} = x^3$ $\qquad$ and $\frac{x^2}{x^4} = x^{2-4} = x^{-2}$ (or $\frac{1}{x^2}$)

❺ When an exponent is itself raised by an exponent, the result is the exponents multiplied.

$(x^3)^2 = x^{3\times 2} = x^6$

❻ When two or more numbers inside brackets are raised by an exponent, the result is every term inside the brackets is raised separately.

$(3xy^3)^2 = 3^2\, y^2\, (x^3)^2 = 9\, x^2\, y^6$

This also applies to roots, which apply to every part inside them.

$\sqrt{9x^4} = \sqrt{9}\,\sqrt{x^4} = 3\, x^2$

Fractions entirely inside brackets to a power also have every part raised to the power.

$\left(\frac{2}{x^2}\right)^3 = \frac{2^3}{(x^2)^3} = \frac{8}{x^6}$

❼ The root of any number can be written as that term to one over the root

$\sqrt[4]{16x^2} = (16x^2)^{\frac{1}{4}}$

This fraction can then be multiplied out as a power in awkward cases.

$\sqrt[3]{27x^2} = (27x^2)^{1/3} = 27^{1/3}\, x^{2\times 1/3} = 3x^{2/3}$

❽ Negative exponents are used to indicate a division by that exponent and **not** that the number is negative. Moving a term from denominator and numerator or vice versa removes the negative (but otherwise leaves the exponent untouched).

$x^{-2.5} = \frac{1}{x^{2.5}}$ $\qquad$ and $\frac{5}{x^{-2}} = 5x^2$

If a negative is applied to all of a fraction, it flips denominator with numerator.

$\left(\frac{2}{x^2}\right)^{-1} = \frac{x^2}{2}$

❾ Items not bracketed are **never** affected by what is happening to other terms.

$5x^{-3} = \frac{5}{x^3}$ $\qquad$ and $\frac{3}{kx^{-1}} = \frac{3x}{k}$