Sequences and Series Practice #2

$t_n = a + (n-1)\,d$ $\qquad$ $t_n = a\,r^{n-1}$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\qquad$ $S_\infty = \frac{a}{1 - r}$

1. A tunnelling company has dug 350 metres on March 1st. On that day they start using new equipment that digs 40 metres per day. They work seven days a week.

   a) How long should the tunnel be on 22 March?

   b) When will they finish the 1750 metre tunnel?

   c) Every day Bill walks in to the machine, and back out again at the end of the day. During the 31 days of March what total distance will he walk in the tunnel?

2. In 2005 a car is worth $35,000. If it is estimated to lose 15% of its value each year, how valuable will it be in 2010?

3. A company is mining a coal seam. In the first month they mine 250 tonnes of coal. Experience has taught them that production tends to be 90% of the previous month (as the mine gets deeper it takes longer to get to the seam and to get the coal out).

   a) How much coal would they expect to get out in the first year?

   b) When will production fall below 100 tonnes per month?

4. Company A has profits of $3.6 Million, and is increasing at 2% each year. Company B has profits of $2.8 Million, but they are increasing at 4% per year. When will the profits of Company B become higher than those of Company A?

5. The sum of the first three terms of an arithmetic sequence is 33, and the fourth term is 15. What is the 19th term?

Answers: Sequences and Series Practice #2

1. a) How long should the tunnel be on 22 March?

$a = 350$, $d = 40$, want t_{22} $t_n = a + (n - 1)d = 350 + (22 - 1) \times 40$

It will be 1190 metres long

b) When will they finish the 1750 metre tunnel?

$a = 350$, $d = 40$, want $t_n \geq 1750$

$t_n = a + (n - 1)d = 350 + (n - 1) \times 40 = 1750$

Solving gives 36. 31 days in March leaves 5 into April. **On April 5th**

c) During the 31 days of March what total distance will he walk in the tunnel?

$a = 700$, $d = 80$, want S_{31} (or $a = 350$, $d = 40$, and we want $2 \times S_{31}$)

$S_{31} = \frac{n}{2}[\ 2a + (n - 1)d] = \frac{31}{2}[\ 2 \times 700 + (31 - 1) \times 80] = 58{,}900$

Bill walks 58.9 km

2. How valuable will it be in 2010?

$a = 35000$, $r = 0.85$, if 2005 = 1 then 2010 = 6, so $n = 6$, need t_6

$t_6 = a\, r^{n-1} = 35000 \times 0.85^{6-1}$ = **$15529.69**

3. a) How much coal would they expect to get out in the first year?

$a = 250$, $r = 0.9$, $n = 12$ $S_{12} = \frac{a(r^n - 1)}{r - 1} = \frac{250 \times (0.9^{12} - 1)}{0.9 - 1}$ = **1794 tonnes**

b) When will production fall below 100 tonnes per month?

$a = 250$, $r = 0.9$, $t_n = 100$ $t_n = a\, r^{n-1}$ so $100 = 250 \times 0.9^{n-1}$

solving (such as using logs) gives n = 9.7 so **10th month will be first under 100t**

4. When will the profits of Company B become higher than those of Company A?.

$a_A = 3.6$, $r_A = 1.02$, $a_B = 2.8$, $r_B = 1.04$, we need n when $t_{nA} = t_{nB}$

$t_n = a\, r^{n-1}$ so $3.6 \times 1.02^{n-1} = 2.8 \times 1.04^{n-1}$

Solving (logs or guess and check) n = 13.94 **From the 14th year on**

5. What is the 19th term?

$t_4 = a + (4 - 1)d = 15$ $\Rightarrow$ $a + 3d = 15$

$S_3 = t_1 + t_2 + t_3 = a + a + 1d + a + 2d = 33$ $\Rightarrow$ $3a + 3d = 33$

Solving simultaneous equations gives a = 9, d = 2 t_{19} = **45**

Achieved = Q1, Q2 and Q3 a). Merit = Q3 b). Excellence = Q4 and Q5.