Level 2 Trigonometry Harder #3
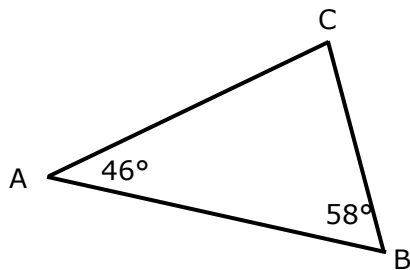
1. Calculate *a*
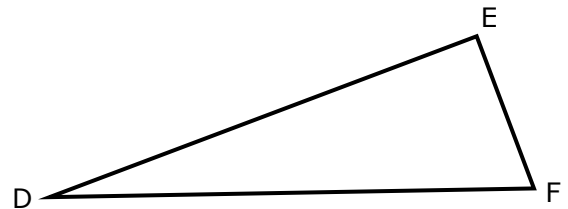
12 →

a

18

8°
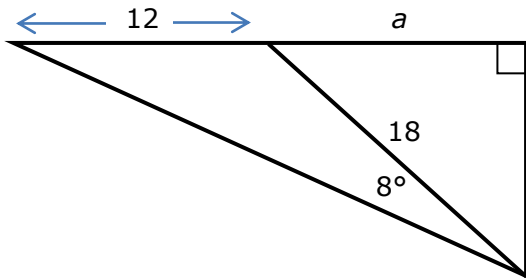
2. What is the perimeter of the quadrilateral?

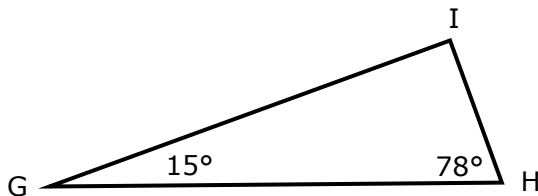29

118°

95°

20

87°
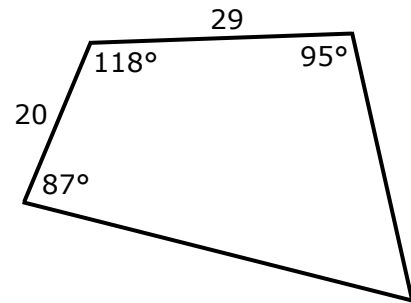
3. The perimeter of this triangle is 60. What are the side lengths?

C

A 46°

58°

B

4. Find the multiplier to the distance along DE and DF that gives a similar triangle of exactly one third the area of the original triangle

E

D

F

5. Find the side lengths if the distance GI + IH = 44.

I

15°

78°

G

H

6. What are **two** sets of possible side lengths so a triangle includes an angle of 38° and has an area of 80 cm²?

38°
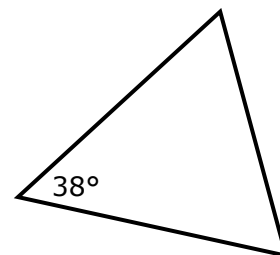
7. The perimeter of this triangle is 50. What are the other side lengths?

P

20

Q

58°

R

8. Calculate *x*°

9

x°

12

18.5

10

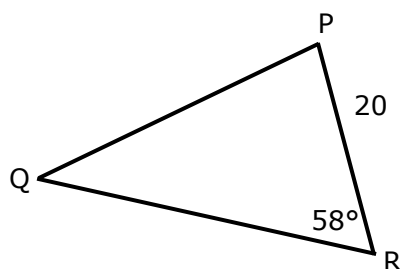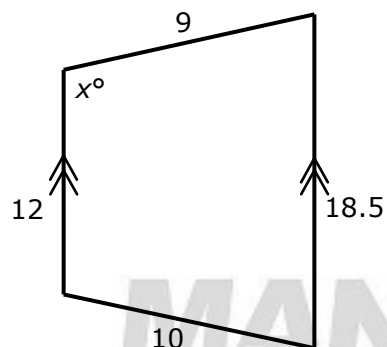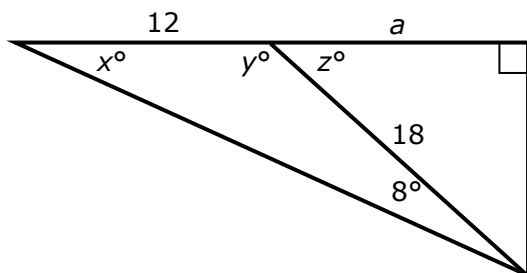## Answers: Level 2 Trigonometry Harder #3

Most of the problems can be approached in more than one way, but the methods given here are usually the shortest.

Rounding errors will occur unless you carry all the decimal places.

1.

Using angles as labelled:

$$\sin x° = \frac{\sin 8°}{12} \times 18 = 0.20876$$

$$x° = \sin^{-1}(0.20876) = 12.05°$$

$$y° = 180° - 8° - 12.05° = 159.95°$$

$$z° = 180° - 159.95° = 20.05°$$

$$a = \cos 20.05° \times 18 = 16.91$$

2. Establishing the values of the top left triangle:

$x^2 = 20^2 + 29^2 - 2 \times 20 \times 29 \times \cos 118° = 1785.59$ $\qquad$ $x = \sqrt{1785.59} = 42.25$

Top right angle: $\sin \theta = \dfrac{\sin 118°}{42.25} \times 20$ $\qquad$ $\theta = 24.70°$ $\qquad$ $95° - 24.70° = 70.30°$

Bottom left angle: $\sin \phi = \dfrac{\sin 118°}{42.25} \times 29$ $\qquad$ $\phi = 37.30°$ $\qquad$ $87° - 37.30° = 49.70°$

So the bottom triangle has a side 42.25 and angles of 70.3°, 49.7° and 60°.

$x = \dfrac{42.25}{\sin 60°} \times \sin 49.7° = 37.21$ $\qquad\qquad$ $y = \dfrac{42.25}{\sin 60°} \times \sin 70.3° = 45.94$

Perimeter is 20 + 29 + 37.21 + 45.94 = 132.15

3. We need to do this via ratios. Make the length AB = 1. ∠ACB = 76° by geometry.

$AC = \dfrac{1}{\sin 76°} \times \sin 58° = 0.8740$ $\qquad\qquad$ $BC = \dfrac{1}{\sin 76°} \times \sin 46° = 0.7414$

The full perimeter is therefore 1 + 0.874 + 0.7414 = 2.6284., Dividing 60 in that ratio:

$60 \times \dfrac{1}{2.6284} = 22.83$ $\qquad$ $60 \times \dfrac{0.874}{2.6284} = 19.95$ $\qquad$ $60 \times \dfrac{0.7414}{2.6284} = 16.92$

Gives side lengths of AB = 22.83, AC = 19.95 and BC = 16.92

4. $½ \times x \times y \times \sin \theta = 3 \times (½ \times m\, x \times m\, y \times \sin \theta)$

$1 = 3m^2$ $\qquad \Rightarrow \qquad$ $m = 0.57735$

5. Let side GI be x and side HI be y: $x + y = 44$, so $y = 44 - x$

$$\frac{x}{\sin 78°} = \frac{y}{\sin 15°} \qquad \Rightarrow \qquad \frac{x}{\sin 78°} = \frac{44 - x}{\sin 15°}$$

$x \sin 15° = 44 \sin 78° - x \sin 78°$ $\qquad$ GI $= \dfrac{44 \sin 78°}{\sin 15° + \sin 78°} = 34.79$ $\qquad$ so HI $= 9.21$

6. Let the sides either side of the angle be the same (so the triangle is isosceles)

$80 = \frac{1}{2} \times x \times x \times \sin 38°$, so the sides are 16.87 each

The side opposite 38°: $d^2 = 16.87^2 + 16.87^2 - 2 \times 16.87 \times 16.87 \times \cos 38°$ $d = 9.92$

So one possible answer is 16.87, 16.87 and 9.92.

For another possible answer, let one side by the angle be a set value. Say 20:

$80 = \frac{1}{2} \times 20 \times x \times \sin 38°$, so the side is 14.23

The side opposite the 38°: $d^2 = 20^2 + 14.23^2 - 2 \times 20 \times 14.23 \times \cos 38°$ $d = 11.48$

So another possible answer is 20, 14.23 and 11.48.

There are an infinite number of actual answers, depending on the starting assumptions.

7. Missing sides can be QP = y and QR = x. $\qquad\qquad$ $y^2 = x^2 + 20^2 - 2 \times x \times 20 \times \cos 58°$
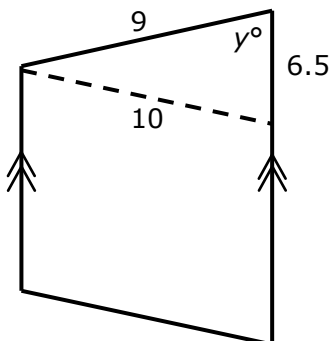
But perimeter is 50 so $x + y + 20 = 50$ $\qquad \Rightarrow y = 30 - x$

$(30 - x)^2 = x^2 + 20^2 - 2 \times x \times 20 \times \cos 58°$

$900 - 60x + x^2 = x^2 + 400 - 21.1968x$ $\qquad\qquad$ which cancels x^2

$500 = 38.80x$ $\qquad\qquad$ $x = 12.89$ $\qquad$ so $y = 17.11$

8.

The parallel sides means we can collapse the trapezium to a triangle, as shown.

$$\cos y° = \frac{9^2 + 6.5^2 - 10^2}{2 \times 9 \times 6.5} = \frac{23.25}{117}$$

$$y° = \cos^{-1}\left(\frac{23.25}{117}\right) = 78.54°$$

$x°$ and $y°$ are co-interior on parallel lines, so $y° = 180° - 78.54° = 101.46°$