Differentiation Practice #1

Differentiate

1 $y = x^3.\ln(x^2)$

2 $y = (x^2 + 7x)^5$

3 $f(x) = \text{cosec } 2x$

4 $y = 5\sin(4x^4)$

5 $f(x) = 8\,e^{\cos x}$

6 $y = \dfrac{x^2}{(5x+4)^2}$

7 $y = t^3 + \cos 5t$

8 $f(x) = \dfrac{e^{2x} - 7x}{x^3}$

9 $f(x) = 4x^2.\cos(x^2)$

10 $g(x) = (x + 2)^4.\sec x + 7\ln x$

11 $y = \cos(x^3 + 12x) - 3$

12 $f(x) = 9x.e^{x+5}$

Answers Differentiation Practice #1

Differentiate — solution — simplified (not required)

1 $y = x^3.\ln(x^2)$ product f.g so $\frac{dy}{dx}$ = f.g' + f'.g and chain rule ln(u) = $\ln(x^2)$

$\frac{dy}{dx} = x^3 \,.\, [\frac{1}{x^2} \,.\, 2x\,] + 3x^2 \,.\, \ln(x^2)$ $= 2x^2 + 3x^2 \,.\, \ln(x^2) = x^2[2 + 3\ln(x^2)]$

2 $y = (x^2 + 7x)^5$ chain rule of $(u)^5$ so $\frac{dy}{dx} = 5u^4 .\ \frac{du}{dx}$

$\frac{dy}{dx} = 5(x^2 + 7x)^4 \,.\, (2x + 7)$

3 $f(x) = \text{cosec } 2x$ chain rule of cosec(u) so $f'(x) = -\,\text{cosec}(u).\cot(u).\ \frac{du}{dx}$

$f'(x) = (-\text{cosec } 2x \cot 2x) \,.\, (2)$ $= {}^-2 \,.\, \text{cosec } 2x \,.\, \cot 2x$

4 $y = 5\sin(4x^4)$ chain rule of 5 sin(u) so $\frac{dy}{dx} = 5\cos(u).\ \frac{du}{dx}$

$\frac{dy}{dx} = 5 \,.\, [\,\cos(4x^4)\,] \,.\, (16x^3)$ $= 80x^3 \,.\, \cos(4x^4)$

5 $f(x) = 8\,e^{\cos x}$ chain rule of $8e^u$ so $f'(x) = 8e^u .\ \frac{du}{dx}$

$f'(x) = 8 \,.\, e^{\cos x} \,.\, (-\sin x)$ $= {}^-8 \sin x \,.\, e^{\cos x}$

6 $y = \frac{x^2}{(5x+4)^2}$ quotient rule where g = $(5x + 4)^2 = u^2$ so g' = $2u.\ \frac{du}{dx}$

$\frac{dy}{dx} = \frac{(5x+4)^2 .2x - x^2.[\,2(5x+4).(5)]}{(5x+4)^4}$ $= \frac{8x}{(5x+4)^3}$ or product via $x^2(5x+4)^{-2}$

or $\frac{dy}{dx} = (x^2) \,.\, [{}^-2(5x + 4)^{-3} \,.\, (5)] + (2x)(5x + 4)^{-2} = {}^-10x^2(5x + 4)^{-3} + 2x(5x + 4)^{-2}$

7 $y = t^3 + \cos 5t$ separated by + done independently, $\frac{d}{dt}\cos(u) = -\sin(u).\ \frac{du}{dt}$

$\frac{dy}{dt} = 3t^2 + [\,(-\sin 5t\,)\ (5)\,]$ $= 3t^2 - 5\sin 5t$

8 $f(x) = \frac{e^{2x} - 7x}{x^3}$ quotient rule where f = $e^{2x} - 7x$ so f' = $(2e^{2x} - 7x)$ and g = x^3

$f'(x) = \frac{x^3.(\,2e^{2x} - 7) - (\,e^{2x} - 7x).3x^2}{x^6}$ $= \frac{e^{2x}\,(2x-3) + 14x}{x^4}$

or $f'(x) = (e^{2x} - 7x) \,.\, ({}^-3x^{-4}) + (2e^{2x} - 7) \,.\, (x^{-3})$ by product rule and g = x^{-2}

9 $f(x) = 4x^2.\cos(x^2)$ product f.g so $f'(x)$ = f.g' + f'.g and g' = $\frac{d}{dx}\cos(u) = -\sin(u).\frac{du}{dx}$

$f'(x) = 4x^2 \,.\, [{}^-\sin(x^2).(2x)] + 8x \,.\, \cos(x^2)$ $= 8x \,.\, \cos(x^2) - 8x^3 .\ \sin(x^2)$

10 $g(x) = (x + 2)^4 .\ \sec x + 7\ln x$ product f.g so $g'(x)$ = f.g' + f'.g and f' = $\frac{d}{dx}u^4 = 4u^3.\frac{du}{dx}$

$g'(x) = (x + 2)^4 .\ [\,\sec x \,.\, \tan x\,] + [\,4(x + 2)^3 \,.\, (1)\,]\,(\sec x\,) + 7\,(\frac{1}{x})$

$= (x + 2)^3 \,.\, [\,(x + 2)\sec x \,.\, \tan x + 4\sec x\,] + \frac{7}{x}$

11 $y = \cos(x^3 + 12x) - 3$ chain rule of cos(u) so $\frac{dy}{dx} = \sin(u).\ \frac{du}{dx}$ and $\frac{d}{dx}(3) = 0$

$\frac{dy}{dt} = [\,-\sin(x^3 + 12x) \,.\, (3x^2 + 12)\,] - 0$ $= -(3x^2 + 12)\sin(x^3 + 12x)$

12 $f(x) = 9x.e^{x+5}$ product f.g so $f'(x)$ = f.g' + f'.g

$f'(x) = 9x \,.\, e^{x+5} \,.\, (1) + 9 \,.\, e^{x+5}$ $= 9\,(x + 1) \,.\, e^{x+5}$