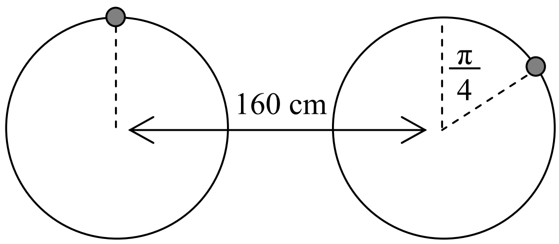
Calculus Trig Identities in Context

1. Write an expression for the sum (addition) of $y = 5 \cos 3x + 7$ and $y = 5 \cos (3x + 4) + 6$ and simplify to a single trig curve.

2. Write the result of multiplying the values of $y = 3 \cos 2x$ by $y = 5 \sin 2x$ as a single trig function.

3. Two wheels of radius 50 cm with their centres 160 cm apart are rotating clockwise at equal speeds of 5 revolutions a minute.



A point is marked on each wheel. At $t = 0$ the left is at the top of the wheel, but the right is rotated $\pi/4$ onwards.

Write an expression for the horizontal distance between the two points and simplify to a single trig expression.

4. The amount of sun's energy hitting an area is the product of the amount per square metre coming from the sun (insolation) and the clarity of the air (the % of sunlight let past).

Both insolation and clarity vary by season.

For a given city, the insolation peaks in January at 5.8 Wm^{-2} and drops to a minimum six months later in July of 1.6.

The clarity of the air peaks in January too at 78% (0.78) and falls to 54% (0.54) six months later.

Model the insolation and clarity as two separate Cosine curves, with time in months.

Multiply the curves together and simplify the result to a sum of simple trig curves. Use that to show that the average energy arriving over the year is 2.568 Wm^{-2}.

Answers: Calculus Trig Identities in Context

1. Sum $= 5\ [\ \cos 3x + \cos (3x + 4)\ ] + 7 + 6$ — grouping like terms

   $= 5\ [\ 2 \cos \frac{(3x)+(3x+4)}{2} \cos \frac{(3x)-(3x+4)}{2}\ ] + 13$ — using "Sums"

   $= 5\ [\ 2 \cos (3x + 2) \cos {}^{-}2\ ] + 13$ — simplifying inside trig

   $\boldsymbol{y = 13 - 4.161 \cos (3x + 2)}$ — as $\cos {}^{-}2$ is a number

2. $3 \cos 2x \times 5 \sin 2x = 15 \cos 2x \sin 2x = 7.5\ [\ 2 \sin 2x \cos 2x\ ] =$ $\boldsymbol{7.5 \sin 4x}$

3. Left is $50 \sin (\frac{2\pi}{12} t)$ and the right is $50 \sin (\frac{2\pi}{12} t - \frac{\pi}{4}) + 160$ relative to the left.

   Difference, which is horizontal distance, is $\Delta = 50 \sin (\frac{2\pi}{12} t - \frac{\pi}{4}) + 160 - 50 \sin (\frac{2\pi}{12} t)$

   $= 50\ [\ \sin (\frac{2\pi}{12} t - \frac{3\pi}{12}) - 50 \sin (\frac{2\pi}{12} t)\ ] + 160$

   $= 50\ [\ \cos (\ ½\ \{\frac{2\pi}{12} t - \frac{3\pi}{12} + \frac{2\pi}{12}\}) \sin (\ ½\ \{\frac{2\pi}{12} t - \frac{3\pi}{12} - \frac{2\pi}{12}\})\ ] + 160$

   $= 50\ [\ \cos (\frac{4\pi}{24} t - \frac{3\pi}{24}) \sin (\frac{-3\pi}{24})\ ] + 160$

   $\boldsymbol{\Delta = 160 - 19.13 \cos (\frac{\pi}{6} t - \frac{\pi}{8})}$ — as $50 \sin (\frac{-3\pi}{24}) = {}^{-}19.13$

4. For the purposes of this I will call January $t = 1$, but it is actually easier as $t = 0$.

   Insolation is $\mathbf{I = 2.1 \cos (\frac{2\pi}{12} (t - 1)) + 3.7}$

   Clarity is $\mathbf{C = 0.12 \cos (\frac{2\pi}{12} (t - 1)) + 0.66}$

   Multiplying $P = C \times I = (2.1 \cos (\frac{2\pi}{12} (t - 1)) + 3.7) \times (0.12 \cos (\frac{2\pi}{12} (t - 1)) + 0.66)$

   $= 0.252 \cos^2(\frac{2\pi}{12} (t - 1)) + 1.386 \cos (\frac{2\pi}{12} (t - 1)) + 0.444 \cos (\frac{2\pi}{12} (t - 1)) + 2.442$

   $= 0.252 \cos^2(\frac{2\pi}{12} (t - 1)) + 1.83 \cos (\frac{2\pi}{12} (t - 1)) + 2.442$

   $= 0.126\ [2 \cos^2(\frac{2\pi}{12} (t - 1))\ ] + 1.83 \cos (\frac{2\pi}{12} (t - 1)) + 2.442$

   $= 0.126\ [\ \cos (\frac{4\pi}{12} (t - 1)) + 1\ ] + 1.83 \cos (\frac{2\pi}{12} (t - 1)) + 2.442$

   $= 0.126 \cos (\frac{4\pi}{12} (t - 1)) + 0.126 + 1.83 \cos (\frac{2\pi}{12} (t - 1)) + 2.442$

   $= 0.126 \cos (\frac{4\pi}{12} (t - 1)) + 1.83 \cos (\frac{2\pi}{12} (t - 1)) + 2.568$

   Since we have two cosine curves – which average themselves to zero over time – the long term average is the constant at the end which they move about = 2.568.

   (Note this is **not** the mid-point between maximum and minimum on the new curve. Although it looks a lot like it if graphed, the result is not a simple trig curve.)