Practice for L3 Distributions #2 – Office Business

Question One

The switchboard in a law office gets an average of 6.4 incoming phone calls during the lunch hour. The company considers reducing the office staff down to one person for that hour.

They know a person can handle up to 12 enquiries during an hour, but then calls start to be missed.

a) What is the probability that during a lunch time that there will be **more** than 12 calls?

b) What is the probability that during a week (five days) that no calls will be missed?

A similar law firm says that it has measured that its staff person at lunchtime is overwhelmed by too many calls on 8% of the lunch hours, but they cannot tell how many calls are missed when this happens.

c) Calculate using the Poisson distribution the mean number of calls missed at a lunch hour by this firm.

Question Two

The law office calculates that it bills its client for a mean of 458 hours every week, with a standard deviation of 38 hours.

a) What is the probability the office will bill for more than 500 hours in a week?

b) It wants to investigate how it manages to get its best results. What value should it set the threshold at for determining the 10% of highest billed weeks?

c) What is the probability that in a particular period of four weeks it will bill a total of less than 1600 hours?

Question Three

A customer satisfaction survey shows that the office's clients are satisfied with the outcome of any court action 72% of the time.

a) Using a binomial distribution, calculate the probability that in a week in which seven court outcomes are reached that at least five of the clients will be satisfied.

b) Explain why a binomial distribution might be considered a good model for part a).

c) What would need to be true to usefully approximate the distribution of successes with a normal distribution rather than binomial.

Calculate the mean and standard deviation for such an approximation.

Answers: Practice for L3 Distributions #2

Q1 a) Poisson distribution with λ = 6.4. Need value for x = 13+. P(13+) = 1 – P(0 – 12)

P(13+ calls) = 0.0143 **= A**

b) Poisson distribution with λ = 6.4. Need value for x = 0 – 12. P(0 – 12) = 0.9857 **= A**

Chance of no missed calls is = 0.9857^5

P(no missed calls) = 0.9305 **= M**

c) Overwhelmed 8%, so the probability not missing any calls is = 0.92.

P(X = x) = $\frac{e^{-\lambda}\lambda^{x}}{x!}$ Here P(0) = 0.92 = $\frac{e^{-\lambda}\lambda^{0}}{0!}$ = $\frac{e^{-\lambda}1}{1}$ = $e^{-\lambda}$.

Log_e both sides: ln(0.92) = ln($e^{-\lambda}$) = –λ ln(e) = –λ. So λ = 0.0.08338 = M

λ is the mean for Poisson.

Mean number calls missed = 0.083 **= E**

Q2 a) Normal distribution with a lower bound of 500 gives **P(bill ≥ 500) = 0.1345**. **= A**

b) Inverse normal, with area 0.1 to right, or 0.9 to left.

Top 10% will exceed 506.7 hours **= M**

c) E(X + Y) = E(X) + E(Y), so in this case we add four means = 4 × 458

Mean billing hours = 1832 hours

Var(X + Y) = Var(X) + Var(Y), so in this case we add 4 variances = 4×38^2

Standard deviation of 4 weeks added = √5776 = 76.

P(4 weeks < 1600) = 0.001134 **= E**

Q3 a) p = 0.72, n = 7, x = 5, 6 or 7. **P(5+) = 0. 6919** **= A**

b) Only two possible results (satisfied or not)

Fixed number of trials (cannot have a fraction of a court case)

Each trial is independent of the other (one judgement will not affect another) 2 = **A**

Probability of success will be constant (not affected by time of year etc.) 3 **= M**

c) You would need a large amount of trials. But since the p = 0.72 of the binomial is not far from 0.5 that number of trials would not need to be huge.

Perhaps 40+ trials would start to give reasonable results (any halfway close number to this would be acceptable, just not less than 20 and no need to get into hundreds) **= M**

For n trials, μ = 0.72n and σ = $\sqrt{n \times 0.72 \times (1 - 0.72)}$ = $\sqrt{0.2016n}$ = $0.45\sqrt{n}$ **= E**