Practice for L3 Distributions #3 – Motorway Traffic

Question One

Modelling predicts a new on-ramp will get a mean of 340 cars per hour, with a standard deviation of 55 cars, at the peak of rush hour. The ramp can only take up to 450 cars per hour.

a) What percentage of days will the on-ramp have too many cars?

Two motorways merge into one at a junction. The Northern motorway has a peak mean flow of 520 cars per hour, with a standard deviation of 102 cars, and the Southern motorway has a peak mean flow of 431 cars per hour, with a standard deviation of 84 cars,

b) Calculate the peak mean flow and standard deviation of the merged traffic at the junction.

c) What is the probability that peak flow on the Southern motorway will exceed that on the Northern?

Question Two

The lights on the motorway are repaired once a month. The probability that a light over will fail and need repair during the month is 0.08.

a) If there are 200 lights at an intersection, what is the most likely number of lights to fail during a month, and what is the probability that number will occur?

b) If there are 200 lights, what is the probability that between 10 and 20 will fail in an month?

c) Calculate part a) using the normal distribution as an approximation. *Show all the values used*.

Question Three

A stretch of motorway is shown to have had 2.25 serious accidents per 10 km stretch in its first year (365 days). Accidents seem to occur at random and a Poisson distribution appears appropriate.

a) Calculate the probability of no serious accidents occurring on a 120 km stretch on a particular day.

b) What is the probability that a particular day during the next year there will be more than a single serious accident on a 80 km stretch?

On the first 80 km stretch there are 32 police chases over the year, which also seems to be a Poisson distribution.

c) What is the probability that there is one accident or police chase, but not both, on a particular day?

Answers: Practice for L3 Distributions #3

Q1 a) Normal distribution with a lower bound of 450 gives **P(cars ≥ 450) = 2.275%**. **= A**

b) E(X + Y) = E(X) + E(Y), so in this case, mean = 520 + 431

Var(X + Y) = Var(X) + Var(Y), so in this case, variance = $102^2 + 84^2$ so σ = √17460.

μ = 951, σ = 132 **= M**

c) E(X − Y) = E(X) − E(Y), so in this case, mean = 520 − 431 = 89

Var(X − Y) = Var(X) + Var(Y), so in this case, σ = √17460 = 132 (as in part b).

S will exceed N when difference between them is < 0, which is P(−∞ < x < 0).

P(S > N) = 0.2501 **= E**

Q2 a) E(X) = np = 200 × 0.08 = 16 is the most likely number to fail.

Binomial distribution, p = 0.08, n = 200, x = 16. **P(16) = 0. 1034** **= A**

b) Binomial distribution, p = 0.08, n = 200, x = 10 – 20.

P(10 – 20) = P(0 – 20) – P(0 – 9) = bcd to 20 – bcd to 9 = 0.8775 – 0.0374

part = **A**

P(10 ≤ *x* ≤ 20) = 0.8401 **= M**

c) Binomial n = 200, p = 0.08, x = 16

so for normal μ = 200 × 0.08 = 16 and σ = $\sqrt{200 \times 0.08 \times (1 - 0.08)}$ = 3.837

Continuity correction means that 16 is represented by 15.5 < x < 16.5 in the normal curve.

P(16) = 0. 1037 **= M**

= E

Q3 a) Poisson distribution with λ = 12 × 2.25 ÷ 365 = 0.07397. Need value for x = 0.

P(0 accidents) = 0.9287 **= A**

b) Poisson distribution with λ = 8 × 2.25 ÷ 365 = 0.04932.

Need value for x = 2+ (not just 2). P(2+) = 1 – P(0) – P(1)

P(2+ accidents) = 0.001177 **= M**

c) Accidents λ = 0.04931. P(0) = 0.9519 P(1) = 0.04694

Chases λ = $\frac{32}{365}$ = 0.08767. P(0) = 0.9161 P(1) = 0.08031

P(1 of either) = P(1 chase) × P(0 accidents) + P(0 chases) × P(1 accident)

P(1 chase or accident) = 0.1194 **= E**