Practice for L3 Equations #4

1. Solve the system of equations

   $x + 2z = 14 + y$ $\quad$ $3x + 4y = 2z - 12$ $\quad$ $2x + 2y + 3z - 15 = 0$

2. Four entrées, four main courses and four desserts cost $280.

   Three entrées, three mains and two desserts cost $195.

   Three entrées cost the same as four desserts.

   What does a dessert cost?

3. Mum, dad and child add up to 80 years old.

   Dad is nine times as old as the child.

   Dad is ten years older than the mum and child added together.

   How much older is the dad than the mum?

4. Bill gets paid $1,352 for forty hours standard time and ten hours overtime, with four hours extra callout bonus.

   The next week he gets $1,224 for forty hours standard, six hours overtime and six hours bonus.

   If overtime is half as much again as standard time, how much is it?

5. Describe fully the nature of the system of equations below:

   $4c + b + 4d = 53$ $\quad$ $2c + 6b + 6d = 86$ $\quad$ $10c + 8b + 14d = 192$

6. Explain, using geometric representations, why the changing of the value of one variable in one of a set of three by three equations gives either 1) an answer where **all** the values are wrong or 2) or will result in a situation that is **inconsistent** (if none of the original equations are the same plane).

   If you want to use a practical example of how it works, use:

   $a + b + c = 6$ $\quad$ $2a + b + c = 7$ $\quad$ $a + 2b + c = 8$

   and alter the c value in the last equation only so that $a + 2b + c = 11$.

Answers: Practice for L3 Equations #4

1. **$x = 2$, $y = {}^{-}2$, $z = 5$**

2.
$4e + 4m + 4d = 280$ | $4e + 4m + 4d = 280$
$3e + 3m + 2d = 195$ | $3e + 3m + 2d = 195$
$3e = 4d$ | $3e - 4d = 0$

$e = 20$, $m = 35$, $d = 15$. Must answer question asked in context. **A dessert costs $15**.

3.
$d + m + c = 80$ | $d + m + c = 80$
$d = 9c$ | $d - 9c = 0$
$d + m + c + 10$ | $d - m - c = 10$

$d = 45$, $m = 30$, $c = 5$. **Dad is 15 years older than the mum.** Answer question asked.

4.
$40s + 10o + 4b = 1352$ | $40s + 10o + 4b = 1352$
$40s + 6o + 6b = 1224$ | $40s + 6o + 6b = 1224$
$o = 1.5s$ | $1.5s - o = 0$

$s = 24$, $o = 36$, $b = 8$. Must answer question asked in context. **Overtime is $36 an hour**.

5. ① $b + 4c + 4d = 53$ ② $6b + 2c + 6d = 86$ ③ $18b + 10c + 14d = 192$

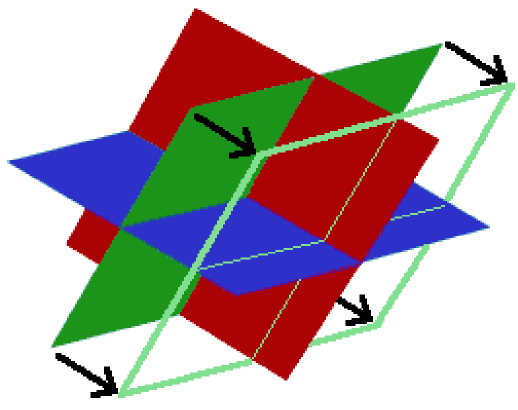
taking 2① + 1② – 1③ gives the equation: 0 = 0 so the system is **dependent**.

There are an **infinite number of solutions**. All three **planes** mutually **intersect along a common line**. Solutions include approximately (10.818, 10.545, 0) and (0, –1.625, 14.875)

6. If we think of the correct solution, the planes represented by the three equations intersect at a point, (1, 2, 3) in our specific case. If one variable is changed then one plane is moved so that it is parallel to what it was previously: $a + 2b + c = 8$ is parallel to $a + 2b + c = 11$. That means the intersection is dragged to that new point, moving both the a and b values.

If the solution is originally dependent then the three planes intersect along a common line, which includes the correct solution. If one plane is moved to a parallel plane, it moves from that common line, generating a system that is inconsistent as it no longer is going through the previous line.

The diagram to the right shows how it works when there is a solution:



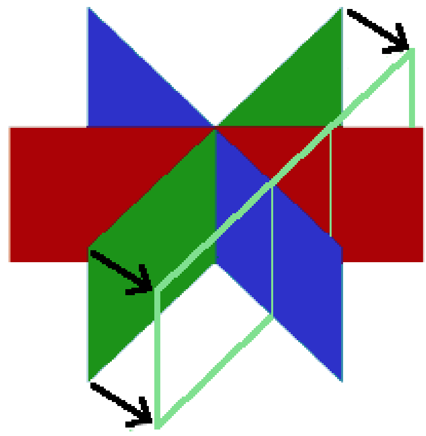
The solid red, green and blue planes intersect at a point in the original, correct, solution.

If the green plane is moved to a parallel plane, because the value of one variable is changed, then we see a movement as shown by the arrows, so we get the light green position for the plane, parallel to the original solid green.

The intersection point – our new solution if we solve – is now at a different place on the red and blue planes too (where the thin light green lines cross). Hence our a and b values are both different if we move any one plane to a parallel one.

Hence **all** values for the new intersection point are incorrect, not just the one altered.

The diagram to the right shows how it works when there is no solution for three different planes.



The solid red, green and blue planes intersect at a common line in the original, correct, solution. The situation is dependent.

If the green plane is moved to a parallel plane, because the value of one variable is changed, then we see a movement as shown by the arrows, so we get the light green position for the plane, parallel to the original solid green.

The intersection points – our new solution if we solve – are different lines on the red and blue planes.

Hence there are no common solutions for the new situation, and the result is inconsistent.

The situation for the original solution being when two planes are the same – a dependent situation – will yield a change that is either dependent (but wrong) or inconsistent, depending which of the planes is moved.