L2 Merit+ Exponents #1

This spans most of the range of harder exponent questions asked. Many can be done in multiple ways.

1. Solve: $8 = \frac{4^{x+1}}{2^{x-2}}$

2. Solve: $1000 \times 1.05^x = 800 \times 1.07^x$

3. Make x the subject of the formula: $p^x = 5^{x-1}$

4. Solve: $9^{x+2} \times 4^x = 17496$

5. Solve: $3 \times 5^{x+2} - 25^x = 1250$

6. Solve: $8 \times 5^x = 7^{x-2}$

7. Fully simplify: $\frac{4^{x+1} + 8^x}{2^{2x}}$

8. Find an expression for x in terms of k if: $\frac{3^{4x+1}}{9^x} = 27^k$

Answers: Merit+ Exponents Practice #1

1. Solve: $8 = \frac{4^{x+1}}{2^{x-2}}$ All are powers of 2, so the easiest method is to use that.

$\Rightarrow 2^3 = \frac{2^{2(x+1)}}{2^{x-2}}$ $\Rightarrow 2^3 \times 2^{x-2} = 2^{2x+2}$ $\Rightarrow 3 + x - 2 = 2x + 2$ $\Rightarrow \boldsymbol{x = -1}$

2. Solve: $1000 \times 1.05^x = 800 \times 1.07^x$ Need to combine the x terms in order to solve.

$\Rightarrow \frac{1000}{800} = \frac{1.07^x}{1.05^x}$ $\Rightarrow 1.25 = \left(\frac{1.07}{1.05}\right)^x$ $\Rightarrow x = \frac{\log 1.25}{\log \frac{1.07}{1.05}}$ $\Rightarrow \boldsymbol{x = 11.826}$

3. Make x the subject of the formula: $p^x = 5^{x-1}$ Need the x in one place.

$\Rightarrow p^x = 5^x \times 5^{-1}$ $\Rightarrow \frac{p^x}{5^x} = \left(\frac{p}{5}\right)^x = 0.2$ $\Rightarrow \boldsymbol{x = \frac{\log 0.2}{\log \frac{p}{5}}}$ (or $x = \frac{\log 5}{\log \frac{5}{p}}$)

Or $p^x = 5^{x-1}$ $\Rightarrow x \log p = (x-1)\log 5 = x \log 5 - \log 5$ $\Rightarrow x \log p - x \log 5 = -\log 5$

$\Rightarrow x(\log p - \log 5) = \log 5$ $\boldsymbol{x = \frac{-\log 5}{\log p - \log 5}}$ (or $x = \frac{\log 5}{\log 5 - \log p}$)

4. Solve: $9^{x+2} \times 4^x = 17496$ Need the x to be alone so we can multiply to combine

$\Rightarrow 9^x \times 9^2 \times 4^x = 17496$ $\Rightarrow 9^x \times 4^x = \frac{17496}{9^2}$ $\Rightarrow 36^x = 216$ $\Rightarrow \boldsymbol{x = 1.5}$

5. Solve: $3 \times 5^{x+2} - 25^x = 1250$ The separation by – into three terms means quadratic.

$\Rightarrow 0 = (5^x)^2 - 75 \times 5^x - 1250$ (as $25^x = 5^x \times 5^x$ and $3 \times 5^{x+2} = 3 \times 5^2 \times 5^x$)

$\Rightarrow (5^x - 25)(5^x - 50) = 0$ $\Rightarrow 5^x = 25$ or $5^x = 50$ $\Rightarrow \boldsymbol{x = 2}$ **or** $\boldsymbol{2.431}$

6. Solve: $8 \times 5^x = 7^{x-2}$ No similar base. Get to simple power of x first, then combine

$8 \times 5^x = 7^x \times 7^{-2}$ $\Rightarrow \frac{8}{7^{-2}} = \frac{7^x}{5^x}$ $\Rightarrow 392 = 1.4^x$ $\Rightarrow \boldsymbol{x = 17.75}$

7. Fully simplify: $\frac{4^{x+1} + 8^x}{2^{2x}}$ Get to powers of two, then find common factors to simplify

$= \frac{2^{2x+2} + 2^{3x}}{2^{2x}}$ $= \frac{2^{2x}(4 + 2^x)}{2^{2x}}$ $= \boldsymbol{4 + 2^x}$

8. Find an expression for x in terms of k if: $\frac{3^{4x+1}}{9^x} = 27^k$ Note all powers of 3

$\Rightarrow \frac{3^{4x+1}}{3^{2x}} = 3^{3k}$ $\Rightarrow 3^{4x+1-2x} = 3^{3k}$ $\Rightarrow 2x + 1 = 3k$ $\boldsymbol{x = \frac{3k-1}{2}}$