L2 Merit+ Exponents #2

1. Solve: $9^x \times 4^{x-2} = 486$

2. Solve: $16^x - 4^{x+1} = 192$

3. Solve: $\frac{25^{x-1}}{5^{x+2}} = 125$

4. Make q the subject of the formula: $512 = 4^{\frac{m}{q}+1}$

5. Solve: $\frac{6^x}{450} = \frac{8^x}{900}$

6. Fully simplify: $\frac{25^x - 5^{x+1}}{125^x - 5^{x+2}}$

7. Solve: $800 \times 3^x = 5^{x+2}$

8. For which values of m can you not solve: $3^{mx-3} = 27^{x^2}$

Answers: Merit+ Exponents Practice #2

1. Solve: $9^x \times 4^{x-2} = 486$ Need the x to be alone so we can multiply to combine

$\Rightarrow 9^x \times 4^x \times 4^{-2} = 486$ $\Rightarrow 9^x \times 4^x = \frac{486}{4^{-2}}$ $\Rightarrow 36^x = 7776$ $\Rightarrow \boldsymbol{x = 2.5}$

2. Solve: $16^x - 4^{x+1} = 192$ The separation by – into three terms means quadratic.

$\Rightarrow (4^x)^2 - 4 \times 4^x - 192 = 0$ (as $16^x = 4^x \times 4^x$ and $4^{x+1} = 4^1 \times 4^x$)

$\Rightarrow (4^x - 16)(4^x + 12) = 0$ $\Rightarrow 4^x = 16$ or $4^x = -12$ (not possible) $\Rightarrow \boldsymbol{x = 2}$

3. Solve: $\frac{25^{x-1}}{5^{x+2}} = 125$ All are powers of 5, so the easiest method is to use that.

$\Rightarrow \frac{5^{2(x-1)}}{5^{x+2}} = 5^3$ $\Rightarrow 5^{2x-2} = 5^3 \times 5^{x+2}$ $\Rightarrow 2x - 2 = 3 + x + 2$ $\Rightarrow \boldsymbol{x = 7}$

4. Make q the subject of the formula: $512 = 4^{\frac{m}{q}+1}$ All base 2 which is the easy way.

$\Rightarrow 2^9 = 2^{\frac{2m}{q}+2}$ $\Rightarrow 9 = \frac{2m}{q} + 2$ $\Rightarrow \boldsymbol{q = \frac{2m}{7}}$

Or, if you log both sides first $512 = 4^{\frac{m}{q}+1}$ $\Rightarrow \log 512 = (\frac{m}{q} + 1)\log 4$

$\Rightarrow \frac{\log 512}{\log 4} = \frac{m}{q} + 1$ $\Rightarrow 4.5 = \frac{m}{q} + 1$ $\Rightarrow \boldsymbol{q = \frac{m}{3.5}}$

5. Solve: $\frac{6^x}{450} = \frac{8^x}{900}$ Need to combine the x terms in order to solve.

$\Rightarrow \frac{900}{450} = \frac{8^x}{6^x}$ $\Rightarrow 2 = (\frac{8}{6})^x$ $\Rightarrow x = \frac{\log 2}{\log \frac{8}{6}}$ $\Rightarrow \boldsymbol{x = 2.409}$

6. Fully simplify: $\frac{25^x - 5^{x+1}}{125^x - 5^{x+2}}$ Get to powers of five, then find common factors to simplify

$= \frac{5^{2x} - 5^x \times 5}{5^{3x} - 5^x \times 25}$ $= \frac{5^x(5^x - 5)}{5^x(5^{2x} - 25)}$ $= \frac{5^x(5^x - 5)}{5^x(5^x - 5)(5^x + 5)}$ $= \boldsymbol{\frac{1}{5^x + 5}}$

7. $800 \times 3^x = 5^{x+2}$ No similar base. Get to simple power of x first, then combine

$800 \times 3^x = 5^x \times 5^2$ $\Rightarrow \frac{3^x}{5^x} = \frac{25}{800}$ $\Rightarrow 0.6^x = 0.03125$ $\Rightarrow \boldsymbol{x = 6.785}$

8. For which values of m can you not solve: $3^{mx-3} = 27^{x^2}$ Note both sides powers of 3

$\Rightarrow 3^{mx-3} = 3^{3x^2}$ $\Rightarrow mx - 3 = 3x^2$ $\Rightarrow 0 = 3x^2 - mx + 3$ $\Delta < 0$ as unsolvable

$\Rightarrow (-m)^2 - 4(3)(3) < 0$ $\Rightarrow m^2 < 36$ $\Rightarrow -6 < m < 6$