Merit and Excellence for Exponents

❶ When multiplying and dividing, the base stays the same or the power stays the same. If the base stays the same the powers are added or subtracted (as you have always done).

$3^x \times 4^x = 12^x$ whereas $4^x \times 4^3 = 4^{x+3}$

$\frac{8^x}{4^x} = \left(\frac{8}{4}\right)^x = 2^x$ whereas $\frac{5^3}{5^x} = 5^{3-x}$

$(3^x)^2 = 9^x = 3^{2x}$

❷ When changing the base, the entire exponent is multiplied.

$9^{x+2} = (3^2)^{x+2} = 3^{2(x+2)} = 3^{2x+4}$

❸ When you want to combine the same variable, you can separate out any numerical portions by the reverse process to combining them.

$3^{t+3} = 3^t \times 9$ and $5^{k-4} = \frac{5^k}{5^4} = \frac{5^k}{625}$

❹ It generally pays to solve using logs at the **end** of the process of solving, after variables have been combined.

Solve $4^{t+3} = 5^t$ $\Rightarrow$ $4^t \times 4^3 = 5^t$

$\Rightarrow$ $64 = \frac{5^t}{4^t} = 1.25^t$

$\Rightarrow$ $t = \frac{\log 64}{\log 1.25} = 18.6377$

❺ Changing the subject works like solving, except it often requires the early use of logs.

Make t the subject of $p^{t+3} = 5^x$ $\Rightarrow$ $(t+3)\log p = x\log 5$

$\Rightarrow$ $t = \frac{x\log 5}{\log p} - 3$

❻ If there are three terms, with + or – separating them, then it is a quadratic unless there is a common factor to take out (see below).

Solve $4^x + 2^{x+2} = 96$ $\Rightarrow (2^x)^2 + 4 \times 2^x - 96 = 0$

$\Rightarrow (2^x - 8)(2^x + 12) = 0$

$\Rightarrow 2^x = 8 \text{ or } 2^x = -12$ $x = 3$

❼ Common factors can be factorised out across + or – to solve or simplify.

Solve $3^x + 3^{x+2} = 2430$ $\Rightarrow 3^x + 3^x \times 9 = 2430$

$\Rightarrow 3^x(1+9) = 2430$

$\Rightarrow 3^x = 243$ $x = 5$

Simplify $\frac{8^x + 4^x}{2^x} = \frac{2^x \times 4^x + 2^x \times 2^x}{2^x} = \frac{2^x(4^x + 2^x)}{2^x} = 4^x + 2^x$