Sequences and Series Practice #3

$t_n = a + (n-1)\,d$ $\qquad$ $t_n = a\,r^{n-1}$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\qquad$ $S_\infty = \frac{a}{1 - r}$

1. Al has just opened a second hand bookshop. In the first month he sells 400 books. In the second months he sells 430. In the third month he sells 460. If that pattern continues:

   a) How many books will he sell in his twentieth month?

   b) How many books will he sell in total in his first two years?

   c) When will he sell his 10,000th book?

2. Sue starts playing Minesweeper on 1 April.

   Her best score on the first day is 880. She gets 10% better each day, so 792 on her next day (90% of her previous score).

   a) What would her score be at the end of the month? (30 days)

   b) How long would it take her to get her best score to under 60?

3. A small gaming company release a new game. In the first month it sells only 300 copies. But each month after that sales increase by 60% over the previous month as players tell others how good it is.

   How many games will it sell in ten months?

4. A disease spreads by personal contact. It lasts a week, and in that time each person with it infects 2.4 other people. If 40 people start infected, how long would it take so that 10,000 people had had the disease?

5. Algal bloom covers 20% of a lake in 2012 and 25% in 2016.

   How much would be covered by 2024, assuming a constant amount of growth?

Answers: Sequences and Series Practice #3

1. a) How many books will he sell in his twentieth month?

$a = 400$, $d = +30$, want t_{20} $t_n = a + (n - 1)d = 400 + (20 - 1) \times 30$

970 books

b) How many books will he sell in total in his first two years?

$a = 400$, $d = +30$, want S_{24}

$S_n = \frac{n}{2}[\ 2a + (n - 1)d] = \frac{24}{2}[\ 2 \times 400 + (24 - 1) \times 30]$

17,880 books

c) When will he sell his 10,000th book?

$a = 400$, $d = +30$, want $S_n = 10000$

$S_n = \frac{n}{2}[\ 2a + (n - 1)d] = \frac{n}{2}[\ 2 \times 400 + (n - 1) \times 30] = 10000$

in his 16th month (solved by quadratics or guess and check)

2. a) What would her score be at the end of the month? (30 days)

$a = 880$, $r = 0.9$, $n = 10$ $t_{30} = a\, r^{n-1} = 880 \times 0.9^{30-1}$ = **41 (rounded)**

b) How long would it take her to get her best score to under 60?

$a = 880$, $r = 0.9$, $n = 10$ $t_n = 880 \times 0.9^{n-1} < 60$

$n < 26.489$ by logs or guess and check

On the 26th day (must be rounded)

3. How many games will it sell in ten months?

$a = 300$, $r = 1.6$, $n = 10$ $S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{300 \times (1.6^{10} - 1)}{1.6 - 1}$ = **54,476 games**

4. If 40 people start infected, how long would it take to spread to a total of 10,000?

$a = 40$ $r = 2.4$ $40 \times 2.5^{n-1} = 10\,000$

$n = 7.3$ by logs or guess and check

During the seventh week

5. How much would be covered by 2024, assuming a constant amount of growth?

$a = 20\%$, $d = \frac{5\%}{4} = 1.25\%$, 2012 is t_0, 2016 is t_{12}

$t_n = a + (n - 1)d$ $t_{12} = 20 + (12 - 1) \times 1.25 = 33.75\%$

Achieved = Q1 a) & b), Q2 a), Q3. Merit = Q1 c) and Q2 b). Excellence = Q4 and Q5.