Sequences and Series Practice #5

$t_n = a + (n-1)\,d$ $\qquad$ $t_n = a\,r^{n-1}$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\qquad$ $S_\infty = \frac{a}{1 - r}$

1. Wendy starts work with a salary of $45,800 for the first year. Each year her salary rises by $2300 more than the previous year.

   a) How much will she be earning in her eighth year?

   b) How much will she earn in total over the first ten years?

2. A bacterial growth starts with 25,000 cells. It increases in number by 8% each hour.

   a) How many cells will there be after 40 hours?

   b) How long does it take the number of cells to double?

3. On the first day of the lockdown Bob spent 2 hours on the computer. He noticed that over the next four days the time increased by a quarter an hour each day. If that continues:

   a) How long will he have spent altogether on the computer over the first fifteen days?

   b) When will he get to spending every hour he is awake (that is, 16) on the computer?

4. A scientist measures that sunlight kills 78% of a particular bacteria in an hour. What percentage of bacteria are left after eight hours in the sunlight?

5. A radioactive sample has a reading of 300 Sieverts. Twenty days later it has a reading of 275 Sv. Estimate its half-life (how long it takes the radiation to fall to half of the start value).

Answers: Sequences and Series Practice #5

1. a) How much will she be earning in her eighth year?

a = 45,800, d = +2300, want t_8 $t_n = a + (n-1)d$ = 45800 + (8 – 1) × 2300

$61,900

b) How much will she earn in total over the first ten years?

a = 45,800, d = +2300, want S_{10} $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 45800 + (10-1) \times 2300]$

$561,500

2. a) How many cells will there be after 40 hours?

a = 25,000, r = 1.08, n = 40 $t_{40} = a\,r^{n-1} = 25000 \times 1.08^{40-1}$

502,882 cells

b) How long does it take the number of cells to double?

a = 25,000, r = 1.08, t_n = 50,000 $t_n = a\,r^{n-1}$ so $50000 = 25000 \times 1.08^{n-1}$

Solving gives **9.0 hours**

3. a) How long will he have spent altogether on the computer over the first fifteen days?

a = 2, d = +0.25, n = 15, want S_{15} $S_{15} = \frac{15}{2}[2 \times 2 + (15-1) \times 0.25]$

56.25 hours

b) When will he get to spending every hour he is awake (that is, 16) on the computer?

a = 2, d = +0.25, t_n = 16, n is wanted $t_n = a + (n-1)d$

$16 = 2 + (n-1) \times 0.25$

Solving gives **57 days**

4. Sunlight kills 78% of bacteria in an hour. What percentage are left after eight hours?

a = 100%, r = 0.22, n = 8 $t_8 = a\,r^{n-1} = 100 \times 0.22^{8-1}$

0.0025%

5. A radioactive sample is 300 Sieverts. Twenty days later it is 275 Sv. Estimate its half-life.

a = 300, r is unknown, n = 20, t_{20} = 275 so $275 = 300 \times r^{20-1}$

Solving gives r = 0.99543 need to find n so that $150 = 300 \times 0.99543^{n-1}$

Solving gives **152 days**

Achieved = Q1 a) & b), Q2 a) & Q3 a). Merit = Q2 b) and Q3 b). Excellence = Q4 & Q5.