Sequences and Series Practice #6

$t_n = a + (n-1)\,d$ $\qquad$ $t_n = a\,r^{n-1}$

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\qquad$ $S_\infty = \frac{a}{1 - r}$

1. On James' 18th birthday his parents started charging him $80 per week board. They promise to put the price up by 30% each year. If he stays at home until he is 25, how much is he going to be paying then?

2. A school enrols 170 new students in 2021. Each year the number enrolling grows by an extra 15 students.

   a) How many students would be expected to enrol in 2029?

   b) What would the total number enrolled from 2021 to 2029?

3. Bunny has $20,000 to invest for eight years. Which of these options is best?

   a) She can get 4% interest (compounding) at the bank.

   b) A Friendly Society offer her $700 in the first year, $750 in the next year, then $800 etc, but only if she leaves it untouched.

4. A real estate company makes its money acting as a rental agent. In their first year they have 90 houses on their books, and they add 35 each year. How long until they have over 400 houses on their books?

5. A company start a new mine. In their first month they dig 150 tons of ore. But it gets harder as they dig down, so each following month they shift 10 tons less than the one before. How much will they have dug in total by the time they can't dig any more?

6. Sandy starts writing a book. On the first day she writes 600 only words, but she gets 20 words better each day than the one before as she gets used to writing. How long would it take her to write a 20,000 word book?

Answers: Sequences and Series Practice #6

1. If he stays at home until he is 25, how much is he going to be paying then?

   a = 25,000, r = 1.3, n = 8 (18 =1, 19 = 2 … 25 = 8) $t_8 = a\,r^{n-1} = 80 \times 1.3^{8-1}$

   $502

2. a) How many students would be expected to enrol in 2029?

   a = 170, d = +15, n = 9, want t_9 $t_n = a + (n-1)d = 170 + (9-1) \times 15$

   290 students

   b) What would the total number enrolled from 2021 to 2029?

   a = 170, d = +15, n = 9, want S_9 $S_n = \frac{n}{2}[2a + (n-1)d]$

   $= \frac{9}{2}[2 \times 170 + (9-1) \times 15]$ = **2,070 students**

3. a) $20,000 to invest for eight years. 4% interest (compounding) at the bank.

   a = 20000, r = 1.04, n = 8 $t_8 = a\,r^{n-1} = 20000 \times 1.04^{8-1}$

   $26,318.63 which is an increase of $**6,318.63**

   b) Friendly Society : $700 in the first year, $750 in the next year, then $800 etc

   a = 20000, d = +50, n = 8 want the sum of those over time = S_8

   $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_8 = \frac{8}{2}[2 \times 700 + (8-1) \times 50]$

   an increase of **$7,000** The Building Society is better over 8 years.

4. How long until they have over 400 houses on their books?

   a = 90, d = +35, n is wanted for $t_n > 400$ $400 > 90 + (n-1) \times 35$

   Solving gives 8.857 **9 years**

5. How much will they have dug in total by the time they can't dig any more?

   a = 150, d = −10, n is wanted so $t_n = 0$ $0 = 150 + (n-1) \times -10$

   Solving gives it runs out after 16 months. Need total by that time = S_{16}

   $S_{16} = \frac{16}{2}[2 \times 150 + (16-1) \times -10]$ = **1200 tons**

6. How long would it take her to write a 20,000 word book?

   a = 600, d = +20, n is wanted so that S_n = 20,000 $S_n = \frac{n}{2}[2a + (n-1)d]$

   $20000 = \frac{n}{2}[2 \times 600 + (n-1) \times 20]$

   Solving gives 24.07 so **25 days**

Achieved = Q1, Q2 a) and b), Q3 a). Merit = Q3 b) and Q4. Excellence = Q5 & Q6.