Practice for L3 Equations #3

1. Solve the system of equations

$5e + g = 2f + 9$ $2e + 2g + 3f + 5 = 0$ $2e + 4f = 3 + g$

2. A squash club has three kinds of membership: super, adult and child.

In January they sell five super, ten adult and one child for $2930. In February they sell one super, three adult and one child for $810. In March they sell four super, five adult and four child for $2120. These can be represented by the following equations:

In April they sell two super, five adult and six child memberships. How much do they raise?

3. A builder buys 100 pavers, 400 bricks and 50 tiles for $1,340.

Another buys 120 pavers, 2,000 bricks and 400 tiles for $4,160.

A brick costs 70 cents more than a tile.

Set up and solve a system of equations to find the cost of pavers, bricks and tiles.

4. Describe fully the nature of the system of equations below:

$2a + 3b + 4c = 31$ $a + 5b + c = 35$ $a - 9b + 5c = 4$

5. Peter sells candy floss at the Easter market in three sizes – Standard, Large and Family.

In 2014 he sold 200 Standard, 200 Large and 100 Family for $1,900.

In 2015 he sold 280 Standard, 240 Large and 80 Family for $2,110.

In 2016 he sold 240 Standard, 220 Large and 90 Family for $2,005.

Did he change the prices he sold the packs at over that time?

6. Consider the following system of equations:

$3x + 2y + 2z = 15$ $x + 2y - z = 12$ $kx = y + z$

Find k so that the system is inconsistent and explain with a full geometrical description.

Answers: Practice for L3 Equations #3

1. $5e - 2f + g =$ $2e + 2g + 3f = -5$ $2e + 4f - g = 3$

   Solution: $e = 2$, $f = {}^{-}1$, $g = {}^{-}3$

2. $5x + 10y + z = 2930$ $x + 3y + z = 810$ $4x + 5y + 4z = 2120$

   Solving gives: $x = 250$, $y = 160$, $z = 80$ $2 \times 250 + 5 \times 180 + 6 \times 80 =$ **$1,880**

3. $100p + 400b + 50t = 1340$ $\Rightarrow$ $100p + 400b + 50t = 1340$

   $120p + 2000b + 400t = 4160$ $\Rightarrow$ $120p + 2000b + 400t = 4160$

   $b = t + 0.7$ $\Rightarrow$ $0p + 1b - 1t = 0.7$

   Solving gives $p = 7$, $b = 1.5$, $t = 0.8$.

   Must answer in context: **pavers cost $7, bricks cost $1.50 and tiles cost 80 cents**.

4. ① $2a + 3b + 4c = 10$ ② $a + 5b + c = 5$ ③ $a - 9b + 5c = 4$

   taking 2① – 3② – 1③ gives the equation: $0 = 1$ so the system is **inconsistent**.

   There are **no solutions**. Taking each pair of planes and finding their line of intersection gives a system of **three parallel lines**, so that you can only solve for two at a time.

5. $200x + 200y + 100z = 1900$ $\Rightarrow$ ① $2x + 2y + z = 19$

   $280x + 240y + 80z = 2110$ $\Rightarrow$ ② $28x + 24y + 8z = 211$

   $240x + 220y + 90z = 2005$ $\Rightarrow$ ③ $240x + 220y + 90z = 2005$

   ② – 8① gives ④ $12x + 8y = 59$ and ③ – 90① gives ⑤ $60x + 40y = 295$. 5④ – ⑤ gives $0 = 0$

   A dependent solution means **prices haven't changed**, because a solution is there (even though we can't figure which one it is). Changing prices would mean an inconsistent result.

6. ① $3x + 2y + 2z = 15$ ② $x + 2y - z = 12$ ③ $k\,x - 1y - 1z = 0$

   The ratio of y and z components of ① –2 × that of 3③. So also for k, giving **$k = {}^{-}1.5$**

   With this value of k the first equation and last equation become parallel, so they never meet. (The equation of ② is irrelevant once you have parallel planes.)