Practice for Merit L3 Probability #2

Question One

A restaurant offers 5 choices of appetizer, 10 choices of main meal and 4 choices of dessert.

A customer can choose to eat just one course, or two different courses, or all three courses.

Assuming all choices are available, how many different possible meals does the restaurant offer?

Question Two

A dentist records the time after the official appointment time the patients have to wait, rounded to the nearest five minutes. The results are recorded below.

Time after	– 5	0	5	10	15	20	25
Frequency	4	5	5	6	3	2	1

a) What was the mean length of any wait after the official appointment time?

b) What is the standard deviation of the waiting?

c) What does the –5 represent in the table, and what is its effect on the calculation of the mean and standard deviation for the wait?

Question Three

There are four soft cover and six hard cover books on a shelf. Four are selected at random.

What is the probability that at least one will be hard cover and at least one will be soft cover?

Question Four

A raffle is being run with 100 tickets, each costing $5.

- first prize is worth $150
- second prize is worth $120
- there are ten consolation prizes of $10

a) Find the expected gain or loss for a person buying a single ticket.

b) What is the variance in the probability distribution? Why is it so large?

c) What should the first prize be in order to make the raffle "fair"?

Question Five

How many ways can the letters ABACUS be arranged so that the vowels (A, A and U) appear together in a row?

Question Six

A bag contains seven marbles marked, –3, –2, –1, 0, 1, 2 and 3.

Two marbles are drawn at random and the numbers on them multiplied together.

What is the probability that the result is a positive number (that is, greater than zero)?

Answers: Practice for Merit L3 Probability #2

1. There are $10 \times 5 \times 4 = 200$ ways of arranging three courses.

   There are $10 \times 5 + 10 \times 4 + 5 \times 4 = 110$ different ways of arranging two courses.

   (appetiser & main, appetiser & desert, main & desert)

   There are $10 + 5 + 4 = 19$ different ways of arranging one course.

   Adding up = 329 different options.

2.

Time after	– 5	0	5	10	15	20	25	Totals
Frequency	3	5	5	6	3	2	1	25
$P(X = x)$	0.12	0.2	0.2	0.24	0.12	0.08	0.04	1
E(X) calc	-0.6	0	1	2.4	1.8	1.6	1	7.2
$E(X^2)$ calc	3	0	5	24	27	32	25	116

   a) $E(X) = 7.2$

   b) $VAR(X) = E(X^2) - E(X)^2 = 116 - 7.2^2 = 64.16$.

   Standard deviation = $\sqrt{Var} = 8.01$

   c) The negative value represents patients being seen **before** their booked time.

   The effect of a negative is to reduce the mean time waited. This might be considered inappropriate, and a weighting value of 0 might be a better indication of average wait. (Giving a mean wait of 7.8).

   The effect of a negative is to increase the variance (and hence standard deviation) because the negative is squared during the calculation, so counts exactly as if +5.

3. There are ${}^{10}C_4 = 210$ ways of selecting four items out of ten.

   4 soft cover and 6 hard cover.

   ${}^4C_3{}^6C_1 = 24$ ways with 3s1h

   ${}^4C_2{}^6C_2 = 90$ ways with 2s2h

   ${}^4C_1{}^6C_3 = 80$ ways with 1s3h

   P(at least one of both types) = $\frac{24+90+80}{210} = 0.9238$

   Note we can check, because there should be 16 missing ways to pick, being

   ${}^4C_4{}^6C_0 = 1$ way with 4s0h

   ${}^4C_0{}^6C_4 = 15$ ways with 0s4h

4.

Outcome, x	– 5	+5	+115	+145	Totals
$P(X = x)$	0.88	0.1	0.01	0.01	1
E(X) calc	–4.4	0.5	1.15	1.45	–1.3
$E(X^2)$ calc	22	2.5	132.25	210.25	367

a) $E(X) = -1.3$ On average the person loses \$1.30.

b) $VAR(X) = E(X^2) - E(X)^2 = 367 - (-1.3)^2 = 365.31$.

The variance is so large because of the disproportionate size of the two big winning values, which are squared in the calculations so have a very large weighting compared to the usual situations of a small loss or gain.

c) $-4.4 + 0.5 + 1.15 + 0.01 \times \text{win} = 0$, so the outcome would need to be +\$275. Plus the cost of the ticket, means the ticket would need to cost \$280.

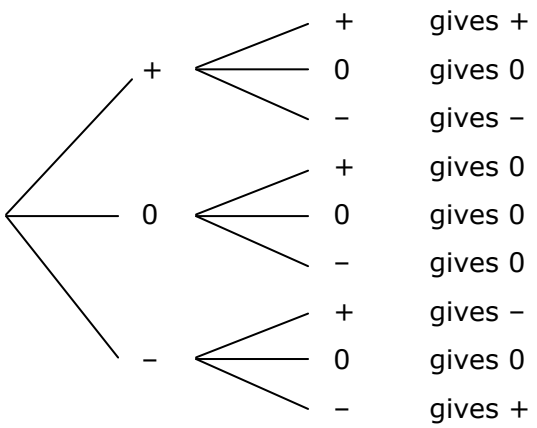
5. If we put the vowels together and treat them as a separate unit, then we have four different elements to organise (AAU) (B) (C) and (S).

Four different elements can be organised in 4! = 24 ways.

However, for each of these the letters AAU can be organised in three different arrangements – AAU, AUA and UAA. So we need to multiply by 3.

There are 72 arrangements.

6.



P(product of 2 is +) $= \frac{12}{42} = \frac{2}{7} = 0.2857$